# APPLICATIONS OF EXP. + LOG FUNCTIONS

**Compound Interest** $A = P\left(1+\frac{r}{n}\right)^{nt}$ OR $A = Pe^{rt}$

continuous compounding

**Exponential Growth** (Business/Human)

$$N = N_0(1 \pm r)^t$$

**Continuous Growth** (Nature)

$$N = N_0 e^{Kt}$$

bacteria
radioactivity

**Newton's Law of Cooling** (Warming)

$$u = T + (u_0 - T)e^{Kt}$$

$u$: final temp of object
$T$: air temp
$u_0$: initial temp of object
$T$: air temp

Safari Sam and Sally return from an expedition deep in the jungle to their small hometown of 2000 people. Immediately following their return, both become ill and are hospitalized with a mysterious and life-threatening jungle fever. After 7 days, a total of 8 people have become ill with the fever. If the CDC is unable to find a treatment for the fever, in how many days will 90% of the town's population have the disease?

$$N = N_0 e^{Kt}$$

Find K.

$$\frac{8}{2} = \frac{2e^{K\cdot 7}}{2}$$

$$\ln 4 = \ln e^{7K}$$

$$\frac{\ln 4}{7} = \frac{7K}{7}$$

$$0.198 = K$$

Solve problem.

$$N = N_0 e^{Kt}$$

$$\frac{1800}{2} = \frac{\cancel{2} e^{0.198t}}{\cancel{2}}$$

$$\ln 900 = \ln e^{0.198t}$$

$$\frac{\ln 900}{0.198} = \frac{0.198t}{0.198}$$

$$34.4 = t$$

$\approx$ 35 days

$$\begin{array}{r} 2000 \\ \times .9 \\ \hline 1800 \end{array}$$

A cup of coffee contains 130 mg of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for half of the caffeine to be eliminated from a person's body?

$$N = N_0 (1 \pm r)^t$$

$$65 = 130(1 - 0.11)^t$$

$$\frac{65}{130} = \frac{\cancel{130}(0.89)^t}{\cancel{130}}$$

$$\log 0.5 = \log (0.89)^t$$

$$\frac{\log(0.5)}{\log(0.89)} = \frac{t \cdot \cancel{\log(0.89)}}{\cancel{\log(0.89)}}$$

$$\boxed{5.9 \text{ hrs.} = t}$$

The Marshalls decided to have a Super Bowl Party. On Saturday, they bought cans of pop to serve at the party; however, they did not have room in their refrigerator for the pop. Since it was February, Bob suggested putting the pop outside to cool. Marilyn was OK with the idea, but wanted to make sure that the pop would not freeze. She took a can of pop from the 72° house to the deck where the air temperature was 25°. After 30 minutes, she checked the can and found that the temperature of the soda had dropped to 60°. How many minutes before the party begins will she want to place all of the pop outside if she wants it to be cooled to 35°?

$$u = T + (u_0 - T)e^{Kt}$$

Solve for K.

$$60 = 25 + (72 - 25)e^{30K}$$

$$60 = 25 + 47e^{30K}$$

$$-25 \quad -25$$

$$\frac{35}{47} = \frac{47e^{30K}}{47}$$

$$\ln\frac{35}{47} = \cancel{\ln} e^{30K}$$

$$\frac{\ln\frac{35}{47}}{30} = \frac{\cancel{30}K}{\cancel{30}}$$

$$-0.00983 = K$$

Find time to cool pop to 35°

$$35^\circ = 25 + (72 - 25)e^{-0.00983t}$$

$$-25 \quad -25$$

$$\frac{10}{47} = \frac{47e^{-0.00983t}}{47}$$

$$\ln\frac{10}{47} = \ln e^{-0.00983t}$$

$$\frac{\ln\left(\frac{10}{47}\right)}{-0.0983} = \frac{-0.00983t}{-0.0098}$$

$$157.4\text{ min} = t$$

$$\approx \boxed{2\text{ hrs } 37\text{ min}}$$