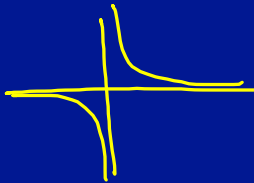
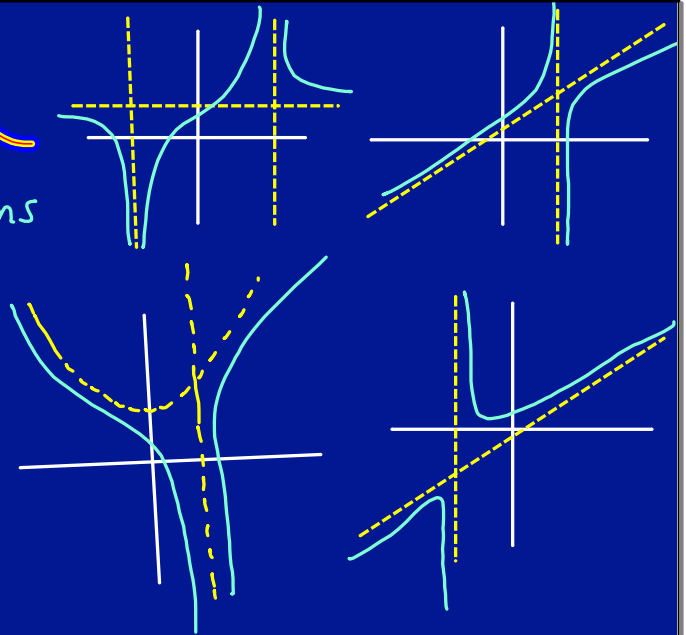


# ASYMPTOTES

→ only with rational functions  
(fractions)

$$y = \frac{1}{x}$$
The graph of the function  $y = \frac{1}{x}$  is shown in yellow. It consists of two hyperbolic branches, one in the first quadrant and one in the third quadrant, separated by the x-axis and y-axis which serve as asymptotes.

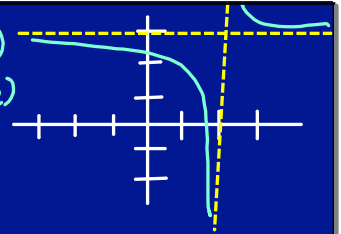


# Vertical Asymptotes

$$\text{Denom} = 0$$

$$y = \frac{1}{x-2} + \frac{3(x-2)}{1(x-2)}$$

$$= \frac{1+3x-6}{x-2}$$



$$y = \frac{3x-5}{x-2}$$

$$\frac{3x}{x} \quad y=3$$

## Horizontal Asymptotes

- 1) Find highest power
- 2) Take that power from both numerator & denominator

$$y = \frac{3x^2+1}{2x^2-8}$$

Vert.  $2x^2-8=0$   
 $2(x^2-4)=0$   
 $2(x+2)(x-2)=0$

$$x=-2; x=2$$

Horiz  
 $\frac{3x^2}{2x^2} = \frac{3}{2}$

$$y = \frac{3}{2}$$

$$y = \frac{4x-1}{2x^2+5}$$

Horiz  
 $\frac{0x^2}{2x^2} = 0$

$$y = 0$$

Vert.  
 $2x^2+5=0$   
 $\sqrt{x^2} = \sqrt{-5/2}$

None

$$y = \frac{3x^3-4x+5}{2x^2+7}$$

Vert.  $2x^2+7=0$   
 $x = \sqrt{-7/2}$

None

Horiz  $\frac{3x^3}{0x^3}$  None

# Slant + Nonlinear Asymptotes (Curvilinear)

Method to find:

Long Division

Numerator has higher power

No Horiz Asymp / will slant/curvilinear

1) Numerator is 1 power higher



2) Numerator is 2 powers higher  
 $y = x^2 - 2x + 3$



3) Numerator is 3 or more powers higher

$$y = x^3$$

$$y = x^3 + x^2 + 2x + 1$$

Find all asymptotes & graph:

- 1) Find asymptotes
- 2)  $x$ - &  $y$ -intercepts
- 3) Plot additional points as needed.

$$f(x) = \frac{4x^2 - 8}{2x - 4}$$

Vertical

$$2x - 4 = 0$$

$$\boxed{x = 2}$$

Horizontal None

$$\begin{array}{r} 2x + 4 \\ 2x - 4 \overline{) 4x^2 + 0x - 8} \\ \underline{-4x^2 + 8x} \phantom{-8} \\ 8x - 8 \end{array}$$

Slant  $y = 2x + 4$

To Graph:

- 1) Find asymptotes
- 2) Find x- + y-int
- 3) Plot additional pts if needed.

$$y = \frac{4x^2 - 8}{2x - 4} \quad (2x - 4) = \frac{4x^2 - 8}{2x - 4} \quad (2x - 4)$$

$\pm 1.4$	0	x-int
0	$\frac{-8}{-4} = 2$	y-int
3	$\frac{28}{2} = 14$	

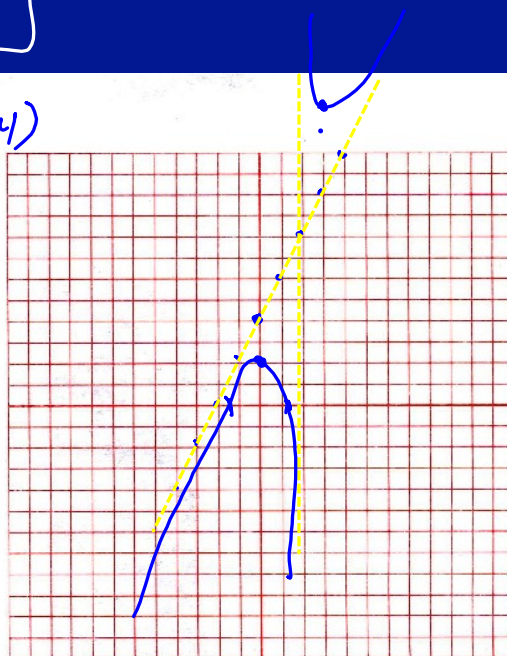
$$0 = 4x^2 - 8$$

$$8 = 4x^2$$

$$\sqrt{2} = \sqrt{x^2}$$

$$\pm\sqrt{2} = x$$

$$\pm 1.4$$



$$f(x) = \frac{5x+10}{x^2-9}$$

Vert.

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

---


$$0 = \frac{5x+10}{\cancel{x^2-9}}$$

$$-10 = 5x$$

$$-2 = x$$

Horiz

$$\frac{0x^2}{x^2-9} = 0$$

$$y =$$

-2	0
0	$\frac{-10}{9}$
2	$\frac{20}{-9} = -4$
4	$\frac{30}{7} = 4\frac{2}{7}$
-4	$\frac{-10}{7} = -1\frac{3}{7}$

