AUGMENTED MATRICES – you have never done this before!

$$4x - 2y = -6$$
$$x + 3y = 11$$

$$\left[\begin{array}{cc|c} 4 & -2 & -6 \\ 1 & 3 & 11 \end{array}\right] \Rightarrow$$

Goal:

$$\left[\begin{array}{cc|c} 1 & 7 & -3 \\ 0 & 1 & 5 \end{array}\right]$$

$$x + 7y = -3$$
$$y = 5$$
$$x + 7(5) = -3$$
$$x = -38$$
$$(-38, 5)$$

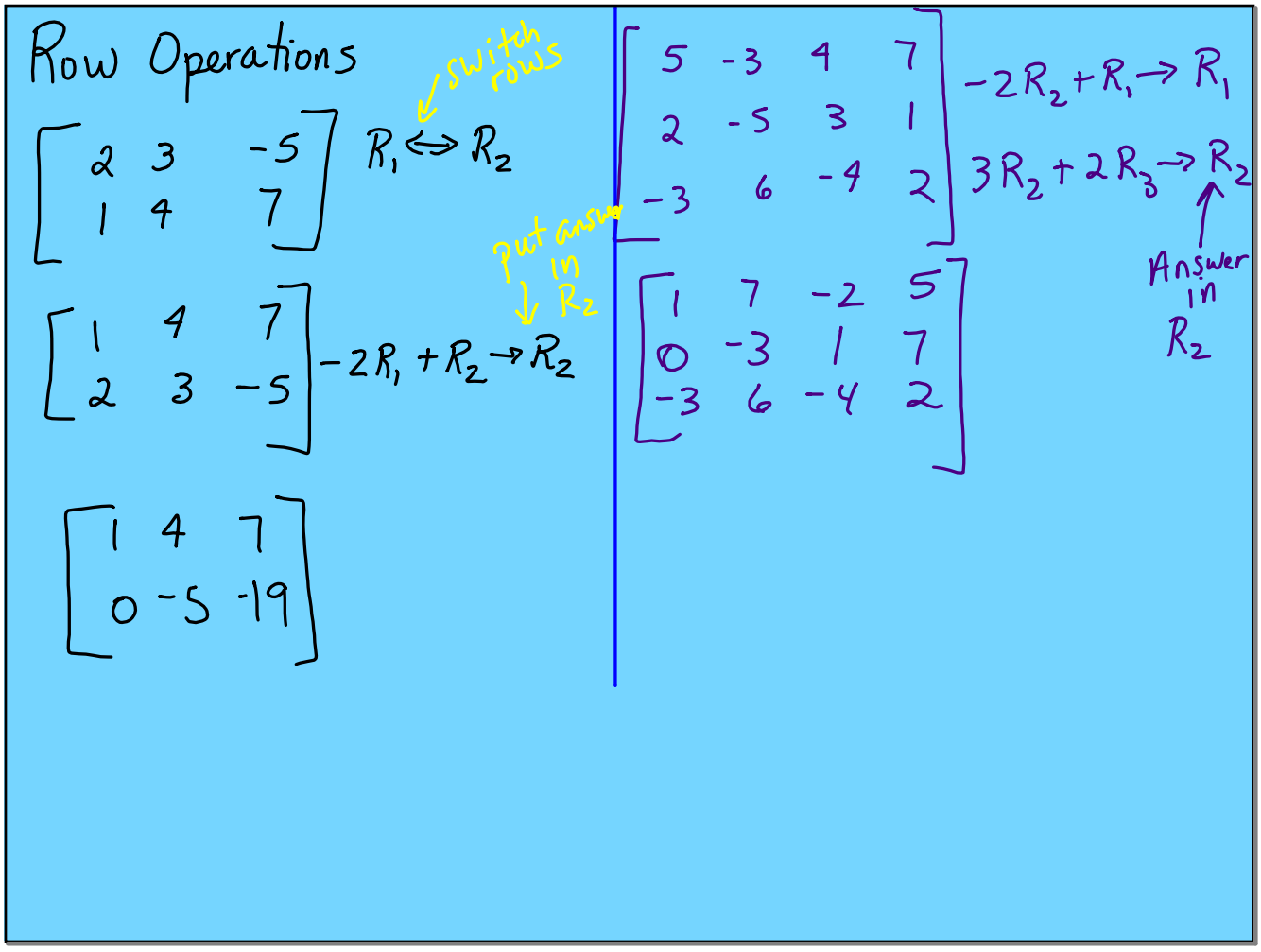
$$x + 2y - 3z = 2$$
$$2x + y + z = 107$$
$$-x - 3y - 2z = 3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 2 \\ 2 & 1 & 1 & 107 \\ -1 & -3 & -2 & 3 \end{array}\right] \Rightarrow$$

Goal:

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 2 \\ 0 & 1 & 5 & -5 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$x + 2y - 3z = 2$$
$$y + 5z = -5$$
$$z = 4$$


Row Operations
switch rows
$R_1 \leftrightarrow R_2$
$-2R_1 + R_2 \to R_2$
put answer in R_2
$-2R_2 + R_1 \to R_1$
$3R_2 + 2R_3 \to R_2$
Answer in R_2

Solve. $7x-2y=-71$, $2x+8y=14$

Goal: $\begin{bmatrix} 1 & \# & \# \\ 0 & 1 & \# \end{bmatrix}$

$\left[\begin{array}{cc|c} 7 & -2 & -71 \\ 2 & 8 & 14 \end{array}\right]$ $R_1-3R_2\to R_1$

Must work downward!

$\left[\begin{array}{cc|c} 1 & -26 & -113 \\ 2 & 8 & 14 \end{array}\right]$ $-2R_1+R_2\to R_2$

$\begin{bmatrix} 1 & -26 & -113 \\ 0 & 60 & 240 \end{bmatrix}$ $\frac{R_2}{60}\to R_2$

$\begin{bmatrix} 1 & -26 & -113 \\ 0 & 1 & 4 \end{bmatrix}$

$x-26y=-113$

$y=4$

$x-26(4)=-113$

$x-104=-113$

$x=-9$

$(-9,4)$

$$x + y - 2z = -1$$
$$4x - y + 3z = 3$$
$$3x + 2y - z = 4$$

Goal: $\begin{bmatrix} 1 & \# & \# & \# \\ 0 & 1 & \# & \# \\ 0 & 0 & 1 & \# \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & -2 & -1 \\ 4 & -1 & 3 & 3 \\ 3 & 2 & -1 & 4 \end{bmatrix} \quad -4R_1 + R_2 \to R_2, \quad -3R_1 + R_3$$

$$\begin{bmatrix} 1 & 1 & -2 & -1 \\ 0 & -5 & 11 & 7 \\ 0 & -1 & 5 & 7 \end{bmatrix} \quad -1R_3 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & -7 \\ 0 & -5 & 11 & 7 \end{bmatrix} \quad 5R_2 + R_3 \to R_3$$

$$\begin{bmatrix} 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & -7 \\ 0 & 0 & -14 & 42 \end{bmatrix} \quad \frac{R_3}{-14} \to R_3$$

$$\begin{bmatrix} 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & -7 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

$$x + y - 2z = -1$$
$$y - 5z = -7$$
$$z = -3$$

$$y - 5(-3) = -7$$
$$y = -22$$

$$x + -22 - 2(-3) = -1$$
$$+6$$
$$x - 16 = -1$$
$$x = 15$$

$$(15, -22, -3)$$

More Tips With Augmented Matrices

$2x - z + 4 = -y$

$2[x - y = \frac{1}{2}z + 1]$

$-4x - 3y + z - 5 = 0$

↓

Get rid of fractions & decimals!

$2x + y - z = -4$

$2x - 2y - z = 2$

$4x - 3y + z = 5$

Legal Row Operations

1) Multiply/Divide a row by a number
2) Add or Subtract two rows
3) Switch position of rows

No!

* Subtract or Add a constant to each number in a row
* Cannot multiply rows together

A vending machine accepts nickels, dimes, and quarters. At the end of a week, there is a total of $536 in the machine. The number of nickels and dimes combines is 360 more than the number of quarters. The number of quarters is 110 more than twice the number of nickels. How many of each type of coin are in the machine?

x = # of nickels
y = # of dimes
z = # of quarters

$$0.05x + 0.10y + 0.25z = 536$$
$$x + y = 360 + z$$
$$z = 110 + 2x$$

$$0.05x + 0.10y + 0.25z = 536$$
$$x + y - z = 360$$
$$-2x + 0y + z = 110$$

$$\left[\begin{array}{ccc|c} 0.05 & 0.10 & 0.25 & 536 \\ 1 & 1 & -1 & 360 \\ -2 & 0 & 1 & 110 \end{array}\right] \quad R_1 \leftrightarrow R_2 \quad 100R_1 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 360 \\ 5 & 10 & 25 & 53600 \\ -2 & 0 & 1 & 110 \end{array}\right] \quad -5R_1 + R_2 \to R_2 \quad 2R_1 + R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 360 \\ 0 & 5 & 30 & 51800 \\ 0 & 2 & -1 & 830 \end{array}\right] \quad \frac{R_2}{5} \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 360 \\ 0 & 1 & 6 & 10360 \\ 0 & 2 & -1 & 830 \end{array}\right] \quad -2R_2 + R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 360 \\ 0 & 1 & 6 & 10360 \\ 0 & 0 & -13 & -19890 \end{array}\right] \quad \frac{R_3}{-13} \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 360 \\ 0 & 1 & 6 & 10360 \\ 0 & 0 & 1 & 1530 \end{array}\right]$$

$x + y - z = 360$
$y + 6z = 10360$

$z = 1530$ quarters

$y + 6(1530) = 10360$
-9180

$y = 1180$ dimes

$x + 1180 - 1530 = 360$
$x - 350 = 360$

$x = 710$ nickels