Exponential Functions

$y = b^x$ ← x: variable; b: constant

$b > 0,\ b \neq 1$

$y > 0 \quad x = \mathbb{R}$

Domain: $\{-\infty, \infty\}$

Range: $(0, \infty)$

$y = 4^{2x+3}$ $\quad y = \left(\frac{1}{3}\right)^{5x}$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$\frac{1}{5^{-3}} = 5^3 = 125$

$36^{1/2} = \sqrt[2]{36^1} = 6$ (2: index)

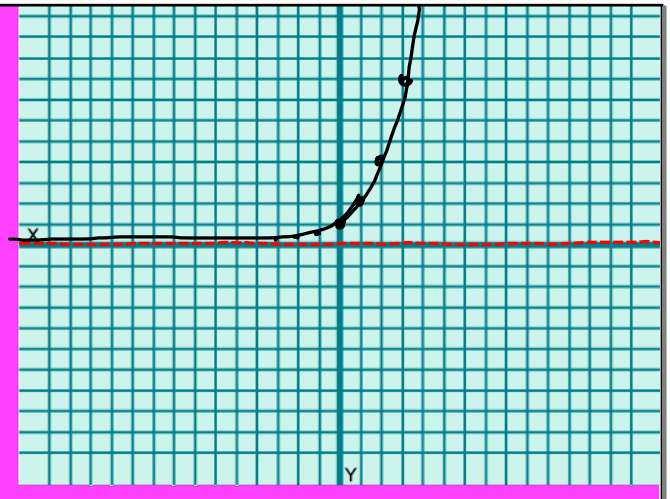
$125^{2/3} = \sqrt[3]{125^2} = 5^2 = 25$

$16^{-3/4} = \frac{1}{\sqrt[4]{16^3}} = \frac{1}{2^3} = \frac{1}{8}$

$y = 2^x$

Exp. Growth

x	y
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
-1	$2^{-1} = \frac{1}{2}$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-3	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

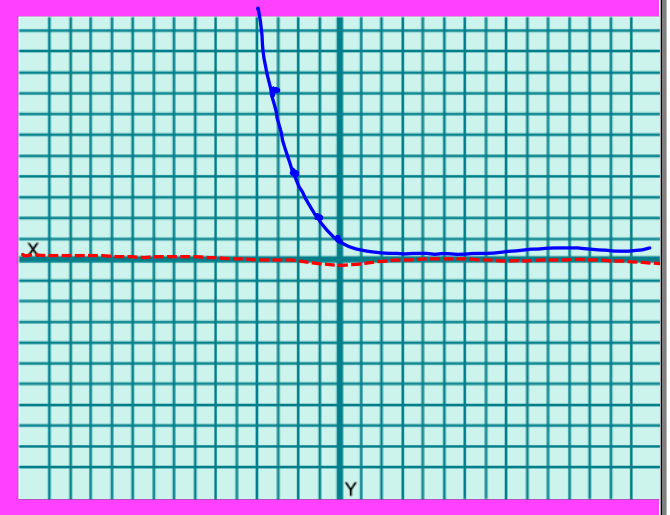


Exponential Decay

Flip over y-axis

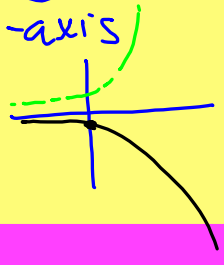
~~$y = -2^x$~~ $y = 2^{-x}$

$y = \left(\frac{1}{2}\right)^x$



Flip over x-axis

$y = -2^x$



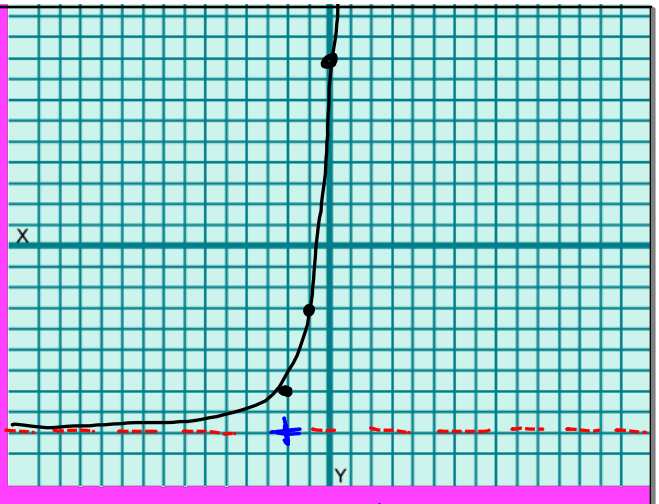
$y = 2 \cdot 3^{x+2} - 9$

Stretch 2 | Left | Down 9

$y = 2x^2$

* by y-word

$y = 3^x$	
0	$3^0 = 1 \cdot 2$
1	$3^1 = 3 \cdot 6$
2	$3^2 = 9 \cdot 18$
~~3~~	~~$3^3 = 27$~~



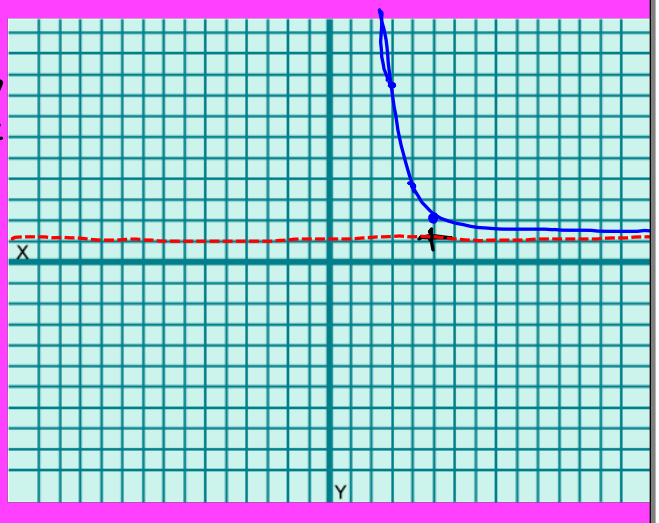
y-axis flip → $-(x-5)$ → $5-x$

$y = e^{5-x} + 1$

Right 5 | Up 1

$y = e^x$	
0	$e^0 = 1$
1	$e^1 = 2.7$
2	$e^2 = 7.4$

$e \approx 2.718$



Basic Exponential Equations

$4^3 = 64$
$4^4 = 256$

Solve.

$144^{x+2} = \left(\frac{1}{12}\right)^{2x-3}$ Make common bases

$\left(12^2\right)^{x+2} = \left(12^{-1}\right)^{2x-3}$

$12^{2x+4} = 12^{-2x+3}$

$2x+4 = -2x+3$

$4x = -1$

$\boxed{x = -\frac{1}{4}}$

$\left(\frac{1}{64}\right)^{5-x} = \left(\sqrt[3]{256}\right)^{x}$

$\left(4^{-3}\right)^{5-x} = \left(\sqrt[3]{4^4}\right)^{x}$

$4^{-15+3x} = \left(4^{4/3}\right)^{x}$

$3\left[-15+3x = \frac{4}{3}x\right]$

$-45+9x = 4x$

$5x = 45$

$\boxed{x = 9}$

Exp Growth/Decay

Human Pop/ Business

$$y = a(1 \pm r)^{t} \leftarrow \text{time}$$

- y: Final Amt
- a: Initial Amt
- r: rate of growth/decay

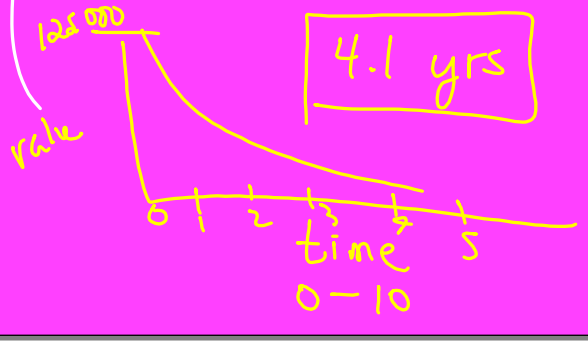
Heavy equipment loses $\frac{1}{5}$ value each year. Earthmover initially cost $125,000.

a) How much is worth after 2 years?

$$y = 125{,}000\left(1 - \frac{1}{5}\right)^{t} \leftarrow 2$$

$$y = 125000\left(\frac{4}{5}\right)^{t}$$

b) When will it be worth $50,000 dollars?



4.1 yrs