MATRIX EQUATIONS – the ultimate method for solving systems of equations.

Inverse Matrix

If $[A] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$[A]^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

!!!

$\frac{8}{8} = 1$

$8 \cdot \frac{1}{8} = 1$

Multiplicative Inverse

$$[B] = \begin{bmatrix} 5 & -6 \\ 3 & -9 \end{bmatrix}$$

$$[B]^{-1} = \frac{1}{-45 - -18}\begin{bmatrix} -9 & +6 \\ -3 & 5 \end{bmatrix}$$

$$= \frac{1}{-27}\begin{bmatrix} -9 & 6 \\ -3 & 5 \end{bmatrix}$$

$-\frac{1}{27} \cdot -27$

$[B]^{-1} * [B]$

$$\frac{1}{-27}\begin{bmatrix} -9 & 6 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & -6 \\ 3 & -9 \end{bmatrix}$$

$$\frac{1}{-27}\begin{bmatrix} -45+18 & 54+-54 \\ -15+15 & 18+-45 \end{bmatrix} = \frac{1}{-27}\begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

← Identity matrix

Solve.

$$2x - 3y = 22$$
$$9x + 4y = +64$$

Must show!

$$[A]^{-1} \cdot \begin{bmatrix} 2 & -3 \\ 9 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ +64 \end{bmatrix} \cdot [A]^{-1}$$

may omit

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{8 - 27} \begin{bmatrix} 4 & +3 \\ -9 & 2 \end{bmatrix} \cdot \begin{bmatrix} 22 \\ +64 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{35} \begin{bmatrix} 88 + +192 \\ -198 + 128 \end{bmatrix}$$

$$= \frac{1}{35} \begin{bmatrix} 280 \\ -70 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ -2 \end{bmatrix}$$

$$(8, -2)$$

$4\left[\frac{3}{4}x - \frac{1}{2}y = 10\right]$ $\quad 3x - 2y = 40$

$20\left[\frac{2}{5}x + \frac{1}{4}y = \frac{21}{2}\right]$ $\quad 8x + 5y = 210$

$$[A]^{-1}\begin{bmatrix} 3 & -2 \\ 8 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 210 \end{bmatrix} \cdot [A^{-1}]$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{15-16}\begin{bmatrix} 5 & 2 \\ -8 & 3 \end{bmatrix} \cdot \begin{bmatrix} 40 \\ 210 \end{bmatrix}$$

$$= \frac{1}{31}\begin{bmatrix} 200+420 \\ -320+630 \end{bmatrix}$$

$$= \frac{1}{31}\begin{bmatrix} 620 \\ 310 \end{bmatrix}$$

$$= \begin{bmatrix} 20 \\ 10 \end{bmatrix}$$

$(20, 10)$

$$27v - 8w + 3x - 7y + 19z = 401.2$$
$$64v - 3w + 18x - 39y - 22z = -3439.3$$
$$v + 91x + 17y + 84z = 3448.4$$
$$8v + 17w - 9x + 33y - 11z = 3028.9$$
$$-6v - 52w - 13x - 29y + z = -4621.5$$

$$\begin{bmatrix} 27 & -8 & 3 & -7 & 19 \\ \vdots & & & & \vdots \\ -6 & -52 & -13 & -29 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 401.2 \\ \vdots \\ -4621.5 \end{bmatrix}$$

Matrix Eq – the only viable method for 4 or more variables.