PARTIAL FRACTIONS

$$\frac{(x+9)7}{(x+9)x-4} + \frac{2(x-4)}{x+9(x-4)} = \frac{7x+63}{(x-4)(x+9)} + \frac{2x-8}{(x-4)(x+9)} = \frac{9x+55}{(x-4)(x+9)}$$

$$(x-4)(x+9)\left[\frac{9x+55}{(x-4)(x+9)}\right] = \frac{A\frac{(x-4)}{(x+9)}}{x-4} + \frac{B\frac{(x-4)}{(x+9)}}{x+9}$$

Get rid of fractions by $*$ by common denom

$$9x+55 = A(x+9) + B(x-4)$$

$$9x+55 = Ax + 9A + Bx - 4B$$

create a system of equations using terms with <u>like powers</u>

$$9 = A + B$$
$$55 = 9A - 4B$$

$$\begin{bmatrix} 1 & 1 \\ 9 & -4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 9 \\ 55 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix} \begin{matrix} A \\ B \end{matrix}$$

$$\boxed{\frac{7}{x-4} + \frac{2}{x+9}}$$

$$\frac{?}{(x+3)(x+2)} = \frac{A}{x^1+3} + \frac{B}{x^1+2}$$

$$\frac{?}{(x^2+4)(x^3+7)} = \frac{Ax^1+B}{x^{②}+4} + \frac{Cx^2+Dx+E}{x^{③}+7}$$

$$\frac{?}{(x-5)^2(x+3)} = \frac{A}{(x^{①}-5)^2} + \frac{B}{x^1-5} + \frac{C}{x^1-3}$$

$$\frac{4}{(x-2)^2} + \frac{7}{x-2} + \frac{8}{x-3}$$

$$(x-2)^2(x-3)$$

$$\frac{}{x^2(x^2-5)} = \frac{A}{x^2} + \frac{B}{x^1} + \frac{Cx+D}{x^2-5}$$

$$(x^1)^2 \qquad (x^1)^2$$

Decompose into partial fractions.

$$\frac{10x^2 + 24x + 8}{x^3 + 3x^2 + 4x + 12}$$

$$x^2(x+3) + 4(x+3)$$

$$(x+3)(x^2+4)$$

$$\frac{10x^2 + 24x + 8}{(x+3)(x^2+4)} = \frac{A}{x+3} + \frac{Bx + C}{x^2 + 4}$$

Get rid fractions!

$$10x^2 + 24x + 8 = A(x^2+4) + (Bx+c)(x+3)$$

$$10x^2 + 24x + 8 = Ax^2 + 4A + Bx^2 + 3Bx + Cx + 3C$$

$$10 = A + B + 0C$$
$$24 = 0A + 3B + C$$
$$8 = 4A + 0B + 3C$$

Use coeff. of like power terms to make system of equations.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \\ 4 & 0 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 10 \\ 24 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 0 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

$$\boxed{\frac{2}{x+3} + \frac{8x}{x^2+4}}$$

$$\frac{4x^2 + 21x + 16}{x^3 + 4x^2 + 4x}$$

$$x(x^2 + 4x + 4)$$

$$x(x+2)(x+2)$$

multiply by com. denom

$$\frac{x}{(x+2)^2}\sqrt{\frac{4x^2 + 21x + 16}{x(x+2)^2}} = \frac{A}{x}^{x(x+2)^2} + \frac{B}{(x+2)^2}^{x(x+2)^2} + \frac{Cx}{x+2}^{x(x+2)^2}$$

$$4x^2 + 21x + 16 = A(x+2)^2 + Bx + Cx(x+2)$$

$$x^2 + 4x + 4$$

$$4x^2 + 21x + 16 = Ax^2 + 4Ax + 4A + Bx + Cx^2 + 2Cx$$

$$4 = A \qquad + C$$
$$21 = 4A + B + 2C$$
$$16 = 4A$$

$$16 = 4A$$
$$4 = A$$

$$4 = A + C$$
$$4 = 4 + C$$
$$0 = C$$

$$21 = 4A + B + 2C$$
$$21 = 16 + B + 0$$
$$5 = B$$

$$\boxed{\frac{4}{x} + \frac{5}{(x+2)^2}}$$