COMPLEX NUMBERS

$a + bi$ $7 + 2i$

↑   ↑
real   imag

$(7 + 2i) + (3 + 4i)$
$= 10 + 6i$

$(5 - 2i)(3 + 4i)$
$15 + 20i - 6i + 8i^2$ $_{-1}$
$= 23 + 14i$
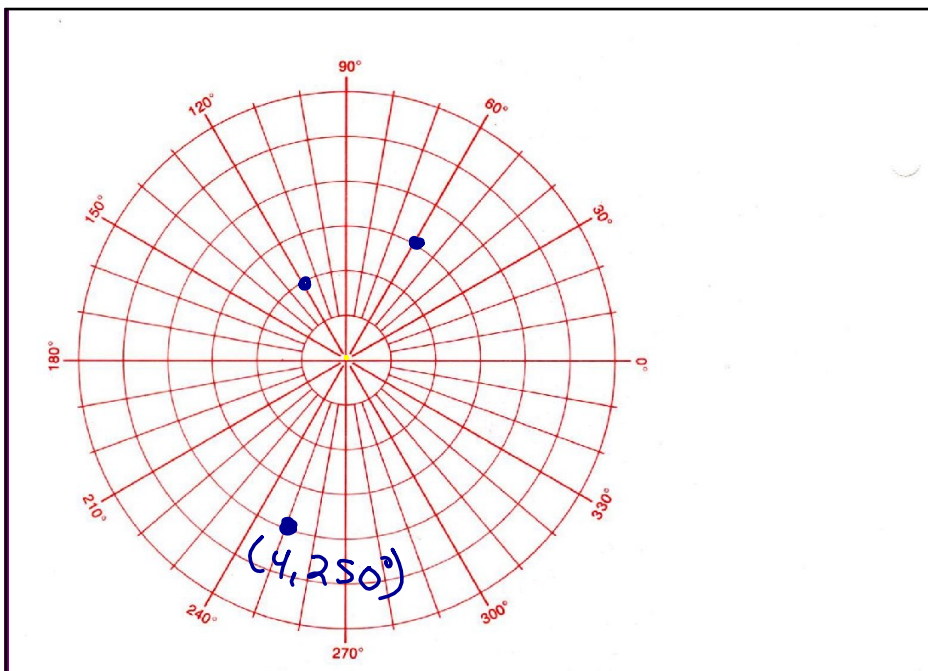
$i = i$ $i = \sqrt{-1}$
$i^2 = -1$
$i^3 = -i$ $i^{83} = i^3 = -i$
$i^4 = 1$ $\frac{83}{4} = 20.75$

$\dfrac{5 + 2i}{4 - 3i} \dfrac{(4 + 3i)}{(4 + 3i)}$

$\dfrac{20 + 15i + 8i + 6i^2}{16 + 9i^2} = \dfrac{14 + 23i}{25}$
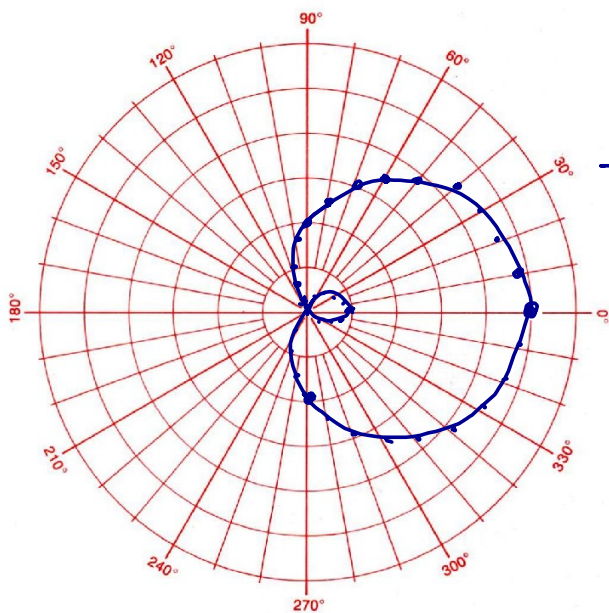
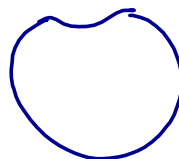$= \dfrac{14}{25} + \dfrac{23i}{25}$

Polar Coordinates

(r, θ)

$(3, 60°)$

$(4, 250°)$

$(-2, 300°) =$
$(2, 120°)$

Limacon

Cardioid

$r = 2 + 3\cos\theta$

θ	r
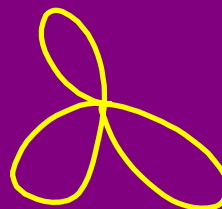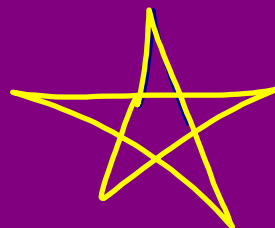
Ultimate Goal

$(5+3i)^8$

$\sqrt{x^7} = \sqrt{7+2i}$

Powers + Roots
of Complex
Numbers
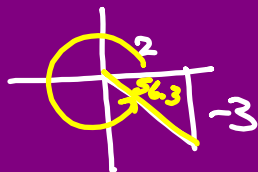
$(5-3\sqrt{2})^8$

$(-3, 0)$

$r=3 \quad \theta = 180°$

$(3, 180°)$

Converting Coordinates

Rectangular
(x, y)

$(2, -3)$

$2^2 + (-3)^2 = r^2$

$\sqrt{13} = \sqrt{r^2}$

$\sqrt{13} = r$

$\left(\sqrt{13}, 303.7°\right)$

Polar
(r, θ)

Rect → Polar

$r = \sqrt{x^2 + y^2}$

$\tan \theta = y/x$

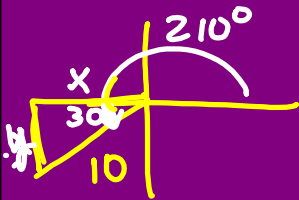$\tan \theta = \dfrac{-3}{2}$

$\theta = \tan^{-1}(3/2)$

$= 56.3°$

$\theta = 360 - 56.3 = 303.7°$

Polar→Rect
$(r, \theta) \longrightarrow (x, y)$

$(10, 210°)$ Find rect. coord.

$\cos 210° = \dfrac{x}{10}$ $\sin 210° = \dfrac{y}{10}$

$10\cos 210° = x$ $10\sin 210° = y$

$10 \cdot \left(-\dfrac{\sqrt{3}}{2}\right) = x$ $10\left(-\frac{1}{2}\right) = y$

$-5\sqrt{3} = x$ $-5 = y$

$\boxed{(-5\sqrt{3}, -5)}$

Convert $-5+5i$ to polar form. of a complex #.

Rectangular Form   Polar Form
$x + yi$ $r\cos\theta + r\sin\theta \cdot i$

$$r(\cos\theta + i\sin\theta)$$

Polar → Rect
$$x = r\cos\theta$$
$$y = r\sin\theta$$
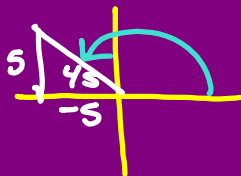
$-\overset{x}{5} + \overset{y}{5}i$ Convert to

Rect → Polar
$$r = \sqrt{x^2 + y^2}$$
$$\tan\theta = y/x$$

$r^2 = 25 + 25$ $\tan\theta = \dfrac{5}{-5} = -1$

$\sqrt{r^2} = \sqrt{50}$

$r = 5\sqrt{2}$ $\theta = 135°$

$$5\sqrt{2}\left(\cos 135° + i\sin 135°\right)$$

$5\sqrt{2}$ cis $135°$

$2(\cos 60° + i\sin 60°)$
Convert to rect.
                form

$2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= 1 + i\sqrt{3}$