August 21, 2018

Exponential & Logarithmic Functions

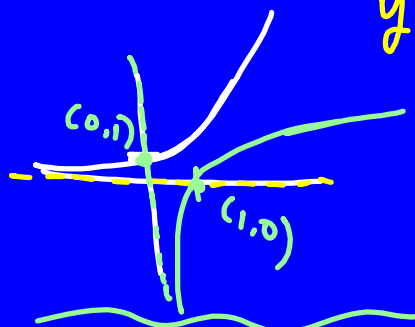
$y = b^x$

$b > 0,\ b \neq 1$

$y = 2^x$

~~$y = (-3)^x$~~

$y = \left(\frac{1}{5}\right)^x$



Exp: $y = b^x$

D: $(-\infty, \infty)$

R: $(0, \infty)$

Log: $y = \log_b x$

D: $(0, \infty)$

R: $(-\infty, \infty)$

$y = \log_6 x$

$\ln 6^y = \ln x$

$y \cdot \ln 6 = \ln x$

$y = \frac{\ln x}{\ln 6}$

$y = \log x$ $\quad b = 10$

$y = \ln x = \log_e x$

Change of Base

$\log_b a = \frac{\ln a}{\ln b}$

$y = \log_8 (x^2 + 3x)$

$y = \frac{\ln (x^2 + 3x)}{\ln 8}$

$\ln a + \ln b = \ln(a \cdot b)$

$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$

$\ln a^p = p \cdot \ln a$

$\ln_2 e^{81} = 81$

$e^{3 \ln 5} = e^{\ln 5^3}$

$= 125$

$\ln \sqrt[5]{e^2} = \ln e^{2/5} = \frac{2}{5}$

$\ln\left(\frac{-3/2}{x+1}\right) + \ln(x-3) = 2\ln x$

$\ln(x^2-2x-3) = \ln x^2$

$x^2-2x-3 = x^2$

$-2x = 3$

$x = -\frac{3}{2}$

No solution

$\ln x - \ln(2x-1) = 8$

$\ln\left(\frac{x}{2x-1}\right) = 8$

$e^{\ln\left(\frac{x}{2x-1}\right)} = e^8$

$\frac{2x-1}{1}\cdot\frac{x}{2x-1} = e^8(2x-1)$

$x = 2e^8x - e^8$

$e^8 = 2e^8x - x$

$e^8 = x(2e^8-1)$

$\frac{e^8}{2e^8-1} = x$

$0.5 \approx x$

$42e^{5x-3} + 9 = 282$

$-9 \quad -9$

$\frac{42e^{5x-3}}{42} = \frac{273}{42}$

$\ln e^{5x-3} = \ln 6.5$

$5x-3 = \ln(6.5)$

$\frac{5x}{5} = \frac{\ln(6.5)+3}{5}$

$x \approx 0.97$

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PARTIAL FRACTIONS

$$\int \frac{26x-43}{6x^2+19x+10} = \frac{A}{3x+2} + \frac{B}{2x+5}$$

$$\frac{26x-43}{(3x+2)(2x+5)} = \frac{(3x+2)}{(3x+2)}\frac{(2x+5)}{(2x+5)}\frac{A}{3x+2} + \frac{(3x+2)}{(2x+5)}\frac{B}{2x+5}$$

$$26x-43 = A(2x+5) + B(3x+2)$$

$$26x-43 = 2Ax + 5A + 3Bx + 2B$$

$$26 = 2A+3B$$
$$-43 = 5A-2B$$

$$\begin{bmatrix} 2 & 3 \\ -5 & -2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 26 \\ -43 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix} \begin{matrix} A \\ B \end{matrix}$$

$$\frac{7}{3x+2} + \frac{4}{2x+5}$$

$$\frac{\sim}{(3x^2+7)(2x-5)} = \frac{Ax+B}{3x^2+7} + \frac{C}{2x-5}$$

$$\frac{\sim}{x^3(x-4)^2} = \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{(x-4)^2} + \frac{E}{(x-4)}$$

$(x')^3 \quad (x')^2$

$\overline{x} \quad \overline{x^2} \quad \overline{x^3}$