CURVE SKETCHING 2

$$f(x) = \frac{x^2+1}{x^2-9}$$

Vertical

$$\lim_{x \to *} f(x) = \pm\infty$$

$$\lim_{x \to 3^+} \frac{x^2+1}{x^2-9} \cdot \frac{\pm}{+} = +\infty$$

Horiz.

$$\lim_{x \to \pm\infty} f(x) = \#$$

$$\lim_{x \to -3^+} \frac{+}{-} = -\infty$$

$$\lim_{x \to \infty} \frac{x^2}{x^2} = 1$$

Vertical $x = 3$
$x = -3$

Horiz $y = 1$

Rel max $(0, -\tfrac{1}{9})$
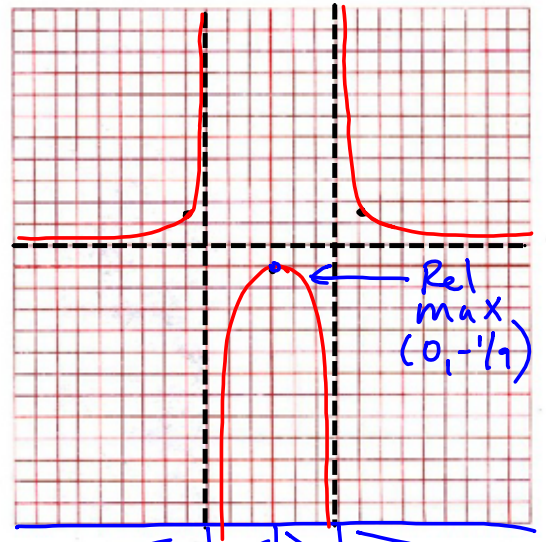
$$f'(x) = \frac{(x^2-9)2x - (x^2+1)2x}{(x^2-9)^2}$$

$$= \frac{2x[x^2-9 - x^2 - 1]}{(x^2-9)^2}$$

0	$-1/9$
4	2.4
-4	2.4

$$0 = \frac{-20x}{(x^2-9)^2} \Longleftarrow$$

$$f''(x) = \frac{(x^2-9)^2 \cdot -20 - (-20x)\cdot 2(x^2-9)\cdot 2x}{(x^2-9)^4}$$

$$0 = -20x$$

$$0 = x$$

$$= \frac{-20(x^2-9)[x^2-9 - x\cdot 4x]}{(x^2-9)^3}$$

(number line)
$\frac{\pm}{+}$ $\frac{+}{+}$ $\frac{-}{+}$ $\frac{-}{+}$
$+$ $+$ $-$ $+$
-3 0 3
-4 -2 1 4

$$= \frac{-20[-3x^2-9]}{(x^2-9)^3} =$$

(number line)
$\frac{\pm}{+}$ $\frac{+}{-}$ $\frac{+}{+}$
$+$ $-$ $+$
-3 3
-4 0 4
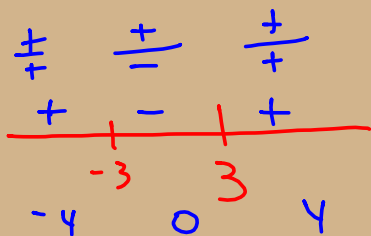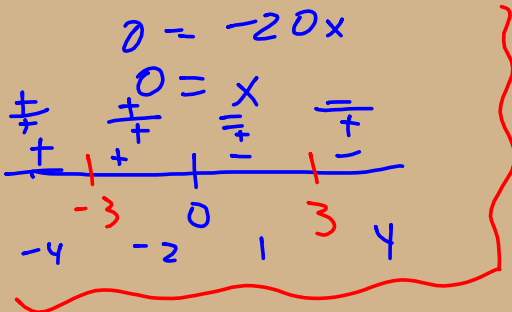
$$\cancel{\frac{60(x^2+3)}{(x^2-9)^3} = 0}$$

$$x^2 + 3 = 0$$
$$\sqrt{x^2} = \sqrt{-3}$$

$$f(x) = x \cdot e^{-2x} = \frac{x}{e^{2x}}$$

Horiz $= y = 0$

$$\lim_{x \to +\infty} \frac{x}{e^{2x}} = \frac{\infty}{\infty}$$

$$\lim_{x \to +\infty} \frac{1}{e^{2x} \cdot 2} = \frac{1}{\infty} = 0$$

$$\lim_{x \to -\infty} \frac{x}{e^{2x}} = \frac{-\infty}{0} = -\infty$$

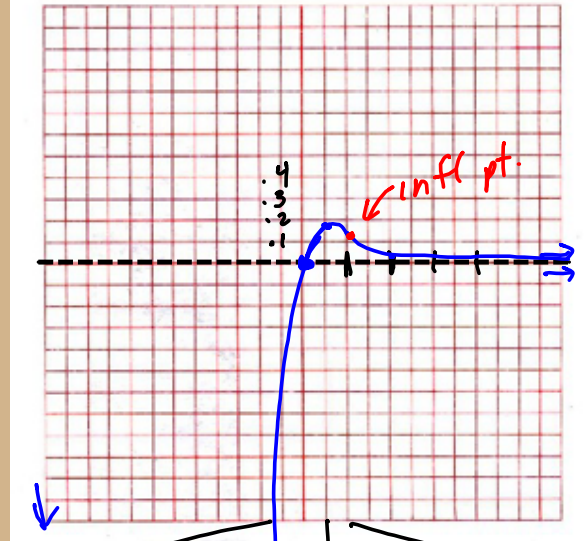- -

$$f'(x) = x \cdot e^{-2x} \cdot -2 + e^{-2x} \cdot 1$$

$$0 = e^{-2x}(-2x+1)$$

+ · +   + · − $x = \frac{1}{2}$

$\dfrac{+ \qquad | \qquad -}{0 \qquad \quad \frac{1}{2} \quad 1}$

− · +   − · −

$\dfrac{- \qquad | \qquad +}{0 \qquad 1 \qquad 2}$

$\frac{1}{2}$	0.18
1	0.13

0	0
2	0.037

$$f''(x) = e^{-2x} \cdot -2 + (-2x+1) \cdot e^{-2x} \cdot -2$$

$$= -2e^{-2x}[1 - 2x + 1]$$

$$\Rightarrow = -2e^{-2x}(2 - 2x)$$

$$= 4e^{-2x}(x-1)$$

$$x = 1$$