4. Use the graph of the equation $y = f'(x)$ in the accompanying figure to find the signs of dy/dx and d^2y/dx^2 at the points A, B, and C.

5. Use the graph of $y = f''(x)$ in the accompanying figure to determine the x-coordinates of all inflection points of f. Explain your reasoning.

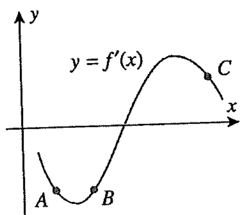


Figure Ex-4

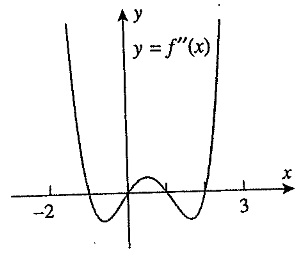


Figure Ex-5

6. Use the graph of $y = f'(x)$ in the accompanying figure to replace the question mark with <, =, or >, as appropriate. Explain your reasoning.

(a) $f(0)$? $f(1)$ (b) $f(1)$? $f(2)$ (c) $f'(0)$? 0
(d) $f'(1)$? 0 (e) $f''(0)$? 0 (f) $f''(2)$? 0

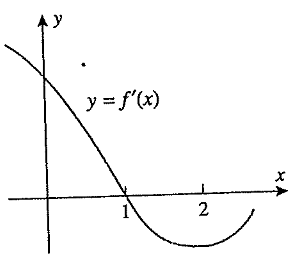


Figure Ex-6

7. In each part, use the graph of $y = f(x)$ in the accompanying figure to find the requested information.
(a) Find the intervals on which f is increasing.
(b) Find the intervals on which f is decreasing.
(c) Find the open intervals on which f is concave up.
(d) Find the open intervals on which f is concave down.
(e) Find all values of x at which f has an inflection point.

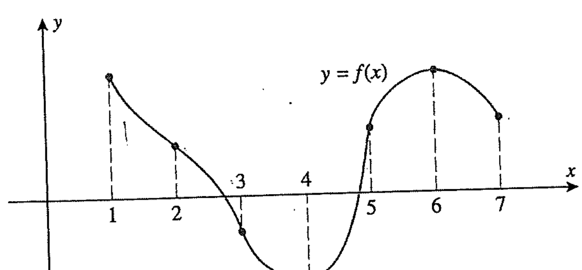


Figure Ex-7

8. Use the graph in Exercise 7 to make a table that shows the signs of f' and f'' over the intervals (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), and (6, 7).

9–10 A sign chart is presented for the first and second derivatives of a function f. Assuming that f is continuous everywhere, find: (a) the intervals on which f is increasing, (b) the intervals on which f is decreasing, (c) the open intervals on which f is concave up, (d) the open intervals on which f is concave down, and (e) the x-coordinates of all inflection points.

9.

INTERVAL	SIGN OF $f'(x)$	SIGN OF $f''(x)$
$x < 1$	–	+
$1 < x < 2$	+	+
$2 < x < 3$	+	–
$3 < x < 4$	–	–
$4 < x$	–	+

10.

INTERVAL	SIGN OF $f'(x)$	SIGN OF $f''(x)$
$x < 1$	+	+
$1 < x < 3$	+	–
$3 < x$	+	+

11–28 Find: (a) the intervals on which f is increasing, (b) the intervals on which f is decreasing, (c) the open intervals on which f is concave up, (d) the open intervals on which f is concave down, and (e) the x-coordinates of all inflection points.

11. $f(x) = x^2 - 3x + 8$
12. $f(x) = 5 - 4x - x^2$
13. $f(x) = (2x + 1)^3$
14. $f(x) = 5 + 12x - x^3$
15. $f(x) = 3x^4 - 4x^3$
16. $f(x) = x^4 - 5x^3 + 9x^2$
17. $f(x) = \dfrac{x - 2}{(x^2 - x + 1)^2}$
18. $f(x) = \dfrac{x}{x^2 + 2}$
19. $f(x) = \sqrt[3]{x^2 + x + 1}$
20. $f(x) = x^{4/3} - x^{1/3}$
21. $f(x) = (x^{2/3} - 1)^2$
22. $f(x) = x^{2/3} - x$
23. $f(x) = e^{-x^2/2}$
24. $f(x) = xe^{x^2}$
25. $f(x) = \ln\sqrt{x^2 + 4}$
26. $f(x) = x^3 \ln x$
27. $f(x) = \tan^{-1}(x^2 - 1)$
28. $f(x) = \sin^{-1} x^{2/3}$

29–34 Analyze the trigonometric function f over the specified interval, stating where f is increasing, decreasing, concave up, and concave down, and stating the x-coordinates of all inflection points. Confirm that your results are consistent with the graph of f generated with a graphing utility.

29. $f(x) = \sin x - \cos x;\ [-\pi, \pi]$
30. $f(x) = \sec x \tan x;\ (-\pi/2, \pi/2)$
31. $f(x) = 1 - \tan(x/2);\ (-\pi, \pi)$
32. $f(x) = 2x + \cot x;\ (0, \pi)$
33. $f(x) = (\sin x + \cos x)^2;\ [-\pi, \pi]$
34. $f(x) = \sin^2 2x;\ [0, \pi]$

EXERCISE SET 5.3 Graphing Utility

1–14 Give a graph of the rational function and label the coordinates of the stationary points and inflection points. Show the horizontal and vertical asymptotes and label them with their equations. Label point(s), if any, where the graph crosses a horizontal asymptote. Check your work with a graphing utility.

1. $\dfrac{2x-6}{4-x}$ **2.** $\dfrac{8}{x^2-4}$ **3.** $\dfrac{x}{x^2-4}$

4. $\dfrac{x^2}{x^2-4}$ **5.** $\dfrac{x^2}{x^2+4}$ **6.** $\dfrac{(x^2-1)^2}{x^4+1}$

7. $\dfrac{x^3+1}{x^3-1}$ **8.** $2-\dfrac{1}{3x^2+x^3}$

9. $\dfrac{4}{x^2}-\dfrac{2}{x}+3$ **10.** $\dfrac{3(x+1)^2}{(x-1)^2}$

11. $\dfrac{(3x+1)^2}{(x-1)^2}$ **12.** $3+\dfrac{x+1}{(x-1)^4}$

13. $\dfrac{x^2+x}{1-x^2}$ **14.** $\dfrac{x^2}{1-x^3}$

15. In each part, make a rough sketch of the graph using asymptotes and appropriate limits but no derivatives. Compare your sketch to that generated with a graphing utility.
(a) $y=\dfrac{3x^2-8}{x^2-4}$ (b) $y=\dfrac{x^2+2x}{x^2-1}$
(c) $y=\dfrac{2x-x^2}{x^2+x-2}$ (d) $y=\dfrac{x^2}{x^2-x-2}$

16. (a) Sketch the graph of
$$y=\frac{1}{(x-a)(x-b)}$$
assuming that $a \neq b$.
(b) Prove that if $a \neq b$, then the function
$$f(x)=\frac{1}{(x-a)(x-b)}$$
is symmetric about the line $x=(a+b)/2$.

17. Show that $y=x+3$ is an oblique asymptote of the graph of $f(x)=x^2/(x-3)$. Sketch the graph of $y=f(x)$ showing this asymptotic behavior.

18. Show that $y=3-x^2$ is a curvilinear asymptote of the graph of $f(x)=(2+3x-x^3)/x$. Sketch the graph of $y=f(x)$ showing this asymptotic behavior.

19–24 Sketch a graph of the rational function and label the coordinates of the stationary points and inflection points. Show the horizontal, vertical, oblique, and curvilinear asymptotes and label them with their equations. Label point(s), if any, where the graph crosses an asymptote. Check your work with a graphing utility.

19. $x^2-\dfrac{1}{x}$ **20.** $\dfrac{x^2-2}{x}$ **21.** $\dfrac{(x-2)^3}{x^2}$

22. $x-\dfrac{1}{x}-\dfrac{1}{x^2}$ **23.** $\dfrac{x^3-4x-8}{x+2}$

24. $\dfrac{x^5}{x^2+1}$

FOCUS ON CONCEPTS

25. In each part, match the function with graphs I–VI without using a graphing utility, and then use a graphing utility to generate the graphs.
(a) $x^{1/3}$ (b) $x^{1/4}$ (c) $x^{1/5}$
(d) $x^{2/5}$ (e) $x^{4/3}$ (f) $x^{-1/3}$

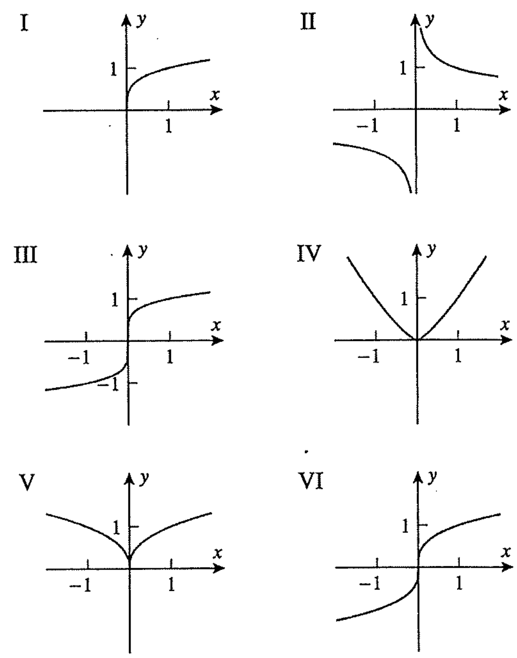


Figure Ex-25

26. Sketch the general shape of the graph of $y=x^{1/n}$, and then explain in words what happens to the shape of the graph as n increases if
(a) n is a positive even integer
(b) n is a positive odd integer.

41–50 Using L'Hôpital's rule (Section 4.4) one can verify that
$$\lim_{x\to+\infty}\frac{e^x}{x}=+\infty,\quad \lim_{x\to+\infty}\frac{x}{e^x}=0,\quad \lim_{x\to-\infty}xe^x=0$$
In these exercises: (a) Use these results, as necessary, to find the limits of $f(x)$ as $x\to+\infty$ and as $x\to-\infty$. (b) Sketch a graph of $f(x)$ and identify all relative extrema, inflection points, and asymptotes (as appropriate). Check your work with a graphing utility.

41. $f(x)=xe^x$ **42.** $f(x)=xe^{-x}$

43. $f(x)=x^2e^{-2x}$ **44.** $f(x)=x^2e^{2x}$

45. $f(x)=x^2e^{-x^2}$ **46.** $f(x)=e^{-1/x^2}$

47. $f(x)=\dfrac{e^x}{1-x}$ **48.** $f(x)=x^{2/3}e^x$

49. $f(x)=x^2e^{1-x}$ **50.** $f(x)=x^3e^{x-1}$

Sec. 5.1 pp. 275-278

14. a) $[-2, 2)$ b) $(-\infty, -2)(2, \infty)$
 c) $(-\infty, 0)$ d) $(0, \infty)$ e) 0

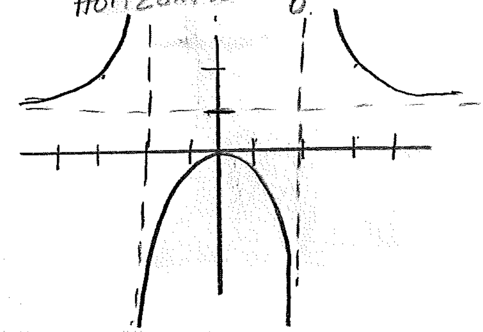
20. a) $(1/4, \infty)$ b) $(-\infty, 1/4)$
 c) $(-\infty, -1/2)\ (0, \infty)$
 d) $(-1/2, 0)$ e) $-1/2,\ 0$

22. a) $(0, 8/27)$ b) $(-\infty, 0)(8/27, \infty)$
 c) none d) $(-\infty, 0)(0, \infty)$
 e) none

24. a) $(-\infty, \infty)$ b) none c) $(0, \infty)$
 d) $(-\infty, 0)$ e) 0

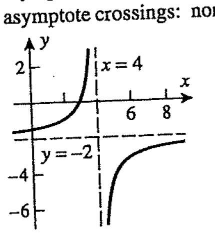
Sec. 5.3 pp. 299-300

4. Vertical: $x = \pm 2$
 Horizontal: $y = 1$

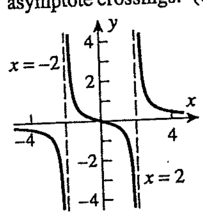


▶ Exercise Set 5.3 (Page 299)

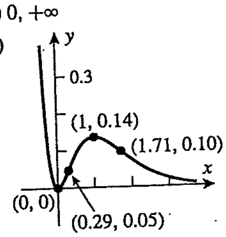
1. stationary points: none;
 inflection points: none;
 asymptotes: $x = 4$, $y = -2$;
 asymptote crossings: none



3. stationary points: none;
 inflection point: (0,0);
 asymptotes: $x = \pm 2$, $y = 0$;
 asymptote crossings: (0,0)



43. **(a)** $0, +\infty$
 (b)



INTERPRETING GRAPHS HANDOUT

Draw the graph of continuous function f having the given properties.

1. $\lim_{x\to+\infty} f(x) = -5$

   $f(1)=1 \quad f(2)=2$

   $(-\infty,1) \; (2,\infty) \quad f'(x)<0$

   $(1,2) \quad f'(x)>0$

   $(-\infty,\infty) \quad f''(x)>0$

2. $\lim_{x\to-4^+} f(x) = +\infty \quad \lim_{x\to+\infty} f(x) = 7$

   $f(1)=2 \quad f(2)=4$

   $(-4,1) \quad f'(x)<0$

   $(1,\infty) \quad f'(x)>0$

   $(-4,2) \quad f''(x)>0$

   $(2,\infty) \quad f''(x)<0$

3. $\lim_{x\to 0} f(x) = -\infty$

   $f(-2)=0 \quad f(2)=1 \quad f(5)=4$

   $(-\infty,0) \; (5,\infty) \quad f'(x)<0$

   $(0,5) \quad f'(x)>0$

   $(-\infty,0) \; (0,2) \; (5,\infty) \quad f''(x)<0$

   $(2,5) \quad f''(x)>0$

4. $\lim_{x\to-4^-} f(x) = \infty \quad \lim_{x\to4^+} f(x) = -\infty$

   $\lim_{x\to\infty} f(x) = 2$

   $f(-8)=6 \quad f(-6)=5 \quad f(-2)=4 \quad f(0)=-1$

   $f(2)=4 \quad f(6)=-1 \quad f(8)=-2$

   $(-\infty,-8) \; (-6,-4) \; (-4,-2)$

   $(0,2) \; (4,6) \; (8,\infty) \quad f'(x)>0$

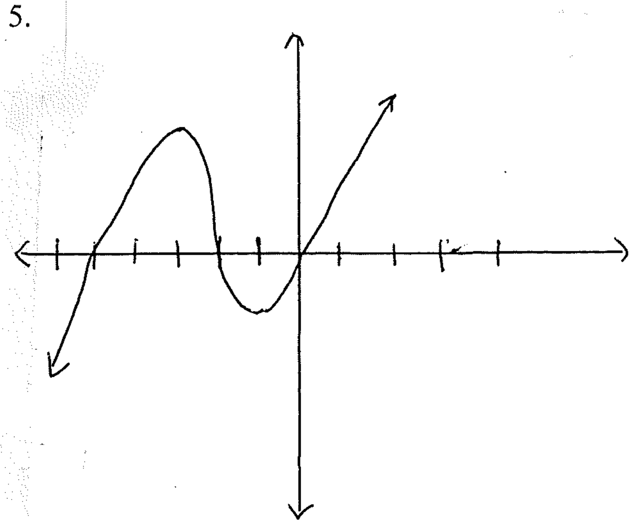
   $(-8,-6) \; (-2,0) \; (2,4) \; (6,8) \quad f'(x)<0$

   $(-4,4) \; (4,\infty) \quad f''(x)<0$

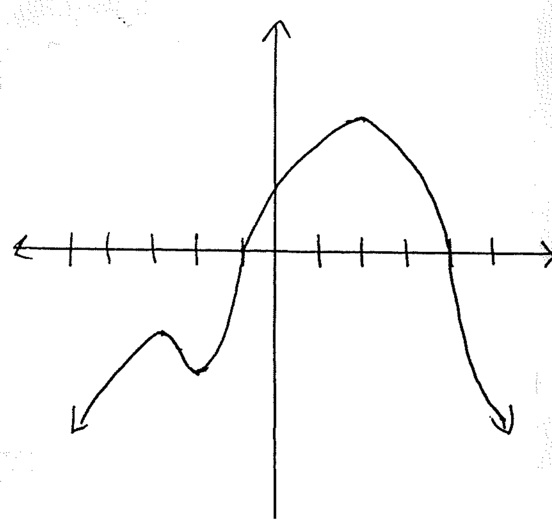
   $(-\infty,-4) \quad f''(x)>0$

Given the following graphs of the derivative of f, identify each of the following: (a) increasing intervals (b) decreasing intervals (c) relative maximums (d) relative minimums (e) concave up intervals (f) concave down intervals (g) inflection points.

5.



6.



Identify all holes, vertical, horizontal, slant, and curvilinear asymptotes of each of the following functions.

7. $f(x)=\dfrac{x^2-2}{x}$

8. $f(x)=\dfrac{x^2+x}{1-x^2}$

9. $f(x)=\dfrac{x^3-4x-8}{x+2}$

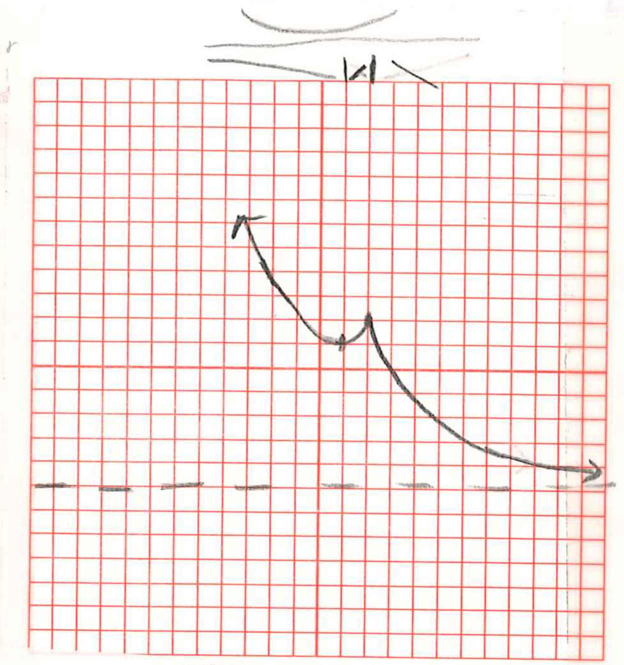
10. $f(x)=\dfrac{2x-5x^2}{x^2+x-2}$

11. $f(x)=\dfrac{x^5}{x^2+1}$

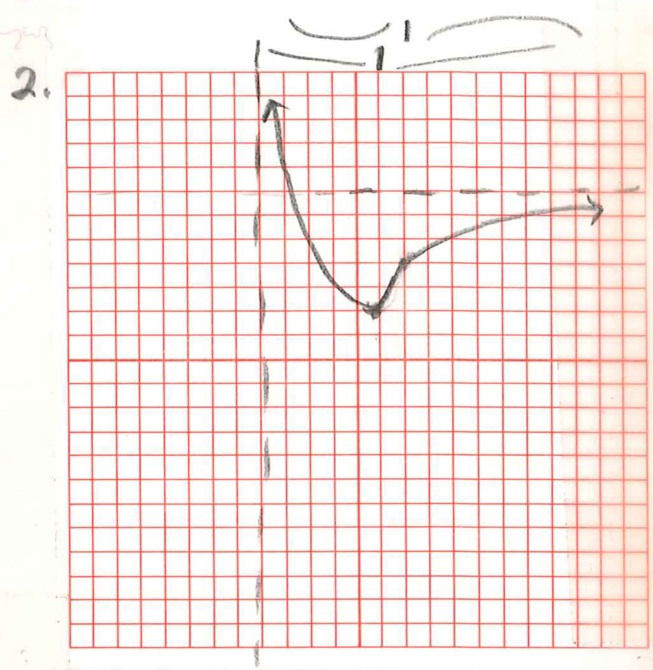
12. $f(x)=\dfrac{4x-3}{x^3-9x}$

ANSWERS

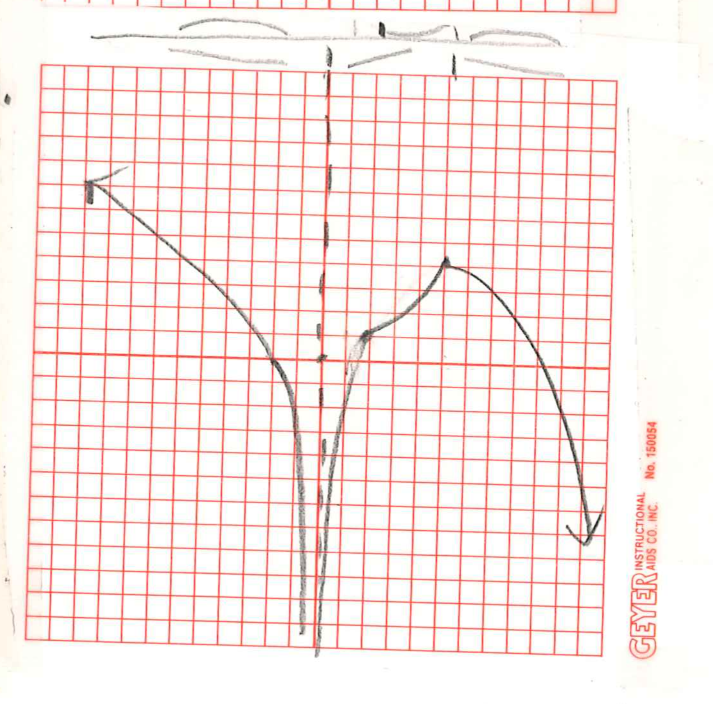
1.

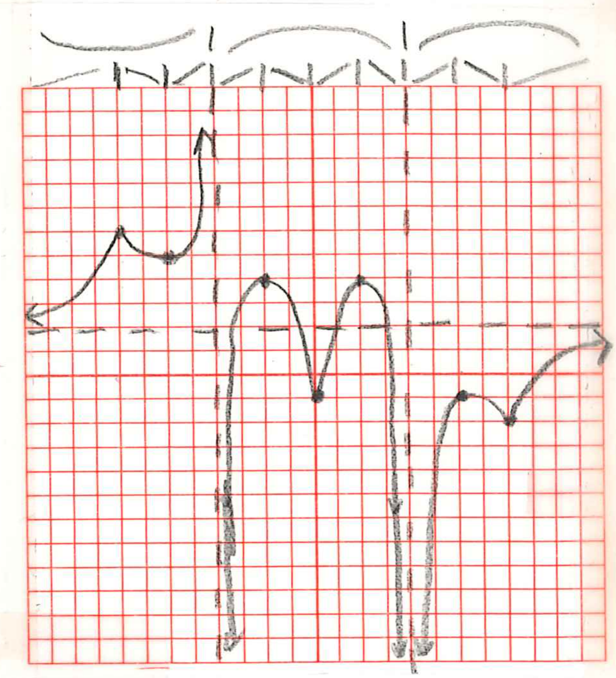


2.



3.





5. (a) $(-5,-2)$ $(0,\infty)$
   (b) $(-\infty,-5)$ $(-2,0)$
   (c) -2
   (d) $-5,\ 0$
   (e) $(-\infty,-3)$ $(-1,\infty)$
   (f) $(-3,-1)$
   (g) $-3,\ -1$

6. (a) $(-1,4)$
   (b) $(-\infty,-1)$ $(4,\infty)$
   (c) 4
   (d) -1
   (e) $(-\infty,-3)$ $(-2,2)$
   (f) $(-3,-2)$ $(2,\infty)$
   (g) $-3,\ -2,\ 2$

7. Vert: $x=0$ Slant: $y=x$

8. Vert: $x=1$ Horiz: $y=-1$ Hole at -1

9. Vert: $x=-2$ Curvilinear: $y=x^2-2x$

10. Vert: $x=-2,\ x=1$ Horiz: $y=-5$

11. Curvilinear: $y=x^3-x$

12. Vert: $x=-3,\ x=0,\ x=3$ Horiz: $y=0$

Assignment:
May omit 2 problems;
must do 10 & 11

CALCULUS WORKSHEET
Absolute Extrema

Identify all absolute extrema on the given interval. Write answers as coordinates.

1. $f(x) = 2x^3 + 3x^2 - 12x;\ [-3,2]$

2. $f(x) = (x^2 + x)^{\frac{2}{3}};\ [-2,3]$

3. $f(x) = \sin x - \cos x;\ [0,\pi]$

4. $f(x) = 4x^3 - 3x^4;\ (-1,2)$

5. $f(x) = x^4 + 4x;\ (-\infty,\infty)$

6. $f(x) = 2x^3 - 6x + 2;\ [0,2)$

7. $f(x) = x^3 - 9x + 1;\ (-\infty,\infty)$

8. $f(x) = \dfrac{x^2+1}{x+1};\ (-5,-1)$

9. $f(x) = 1 + \dfrac{1}{x};\ (0,\infty)$

10. $f(x) = x^3 e^{-2x};\ [1,4]$

11. $f(x) = \dfrac{\ln(2x)}{x};\ [1,e]$

ANSWERS

1. Abs. min. (1,-7); Abs. max. (-2,20)
2. Abs. min. (0,0), (-1,0); Abs. max. $(3,\sqrt[3]{144})$
3. Abs. min. (0,-1); Abs. max. $\left(\dfrac{3\pi}{4},\sqrt{2}\right)$
4. No abs. min.; Abs. max. (1,1)
5. Abs. min. (-1,-3); No abs. max.
6. Abs. min. (1,-2); No abs. max.
7. No absolute extrema
8. No abs. min.; Abs. max. (-2.414,-4.828)
9. No absolute extrema
10. Abs. min $\left(4,64e^{-8}\right)$; Abs. max. $\left(\dfrac{3}{2},\dfrac{27}{8}e^{-3}\right)$
11. Abs. min. $\left(e,\dfrac{\ln 2e}{e}\right)$; Abs. max. $\left(\dfrac{e}{2},\dfrac{2}{e}\right)$

CURVE SKETCHING WITH CAS

For each problem below, complete the following steps. Show work for all steps including the results generated by your calculator on a separate sheet of paper.

1. Find all horizontal, vertical, slant, and curvilinear asymptotes.
2. Find the intervals where the function is increasing and decreasing.
3. Find the intervals where the function is concave up and concave down.
4. Sketch the graph on paper and then graph it on your calculator.
5. Identify all relative and absolute extrema and points of inflection as ordered pairs (x,y).

Problem #1: $f(x)=\dfrac{x^2+1}{x}$

Problem #2: $f(x)=\dfrac{2x^2}{x^2-16}$

Problem #3: $f(x)=\dfrac{x^3-x^2-8}{x-1}$

Problem #4: $f(x)=\dfrac{xe^{-x}}{x+1}$

(Use large sheet of graph paper with portrait orientation. Count large boxes by 1's on the x-axis. Count large boxes by 2's on the y-axis. Make the x-axis the second line up from the bottom.)

CHECK VALUES

PROBLEM #1
Asymptotes: $x = 0$; $y = x$
Decreasing: $(-1,0)$ $(0,1)$
Concave up: $(0, \infty)$

PROBLEM #2
Asymptotes: $x = -4$; $x = 4$; $y = 2$
Decreasing: $(-\infty, 4)$ $(4,0)$
Concave down: $(-4,4)$

PROBLEM #3
Asymptotes: $x = 1$; $y = x^2$
Increasing: $(-1,1)$ $(1,\infty)$
Concave up: $(-\infty,1)$ $(3,\infty)$

PROBLEM #4
Asymptotes: $x = -1$; $y = 0$
Increasing: $\left(\frac{-1-\sqrt{5}}{2},-1\right)$ $\left(-1,\frac{-1+\sqrt{5}}{2}\right)$
Concave down: $(-1,1.27)$