MORE INTEGRATION

$$\int 5x\sqrt{2x+3}\,dx$$

$u = 2x+3 \qquad \frac{u-3}{2} = \frac{\cancel{2}x}{\cancel{2}}$

$du = 2\,dx$

$\frac{du}{2} = dx$

$$\int 5x \cdot u^{1/2} \cdot \frac{du}{2}$$

$$\frac{5}{2}\int \frac{u-3}{2} \cdot u^{1/2} \cdot du$$

$$\frac{5}{4}\int \left(u^{3/2} - 3u^{1/2}\right) du$$

$$\frac{5}{4}\left[\frac{2}{5}u^{5/2} - \frac{2\cdot\cancel{3}}{\cancel{3}}u^{3/2}\right] + C$$

$$\frac{1}{2}u^{5/2} - \frac{5}{2}u^{3/2} + C$$

$$\frac{1}{2}(2x+3)^{5/2} - \frac{5}{2}(2x+3)^{3/2} + C$$

$$\int (x+2)^2 \sqrt{1+x}\, dx$$

$u = 1+x \qquad u-1 = x$

$du = dx$

$$\int (x+2)^2 u^{1/2}\, du$$

$$\int (u-1+2)^2 u^{1/2}\, du$$

$$\int (u+1)^2 u^{1/2}\, du$$

$$\int (u^2+2u+1)\, u^{1/2}\, du$$

$$\int \left(u^{5/2} + 2u^{3/2} + u^{1/2}\right) du$$

$$= \frac{2}{7}u^{7/2} + \frac{2}{5}\cdot 2u^{5/2} + \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{7}(1+x)^{7/2} + \frac{4}{5}(1+x)^{5/2} + \frac{2}{3}(1+x)^{3/2} + C$$

$$\int \tan^8 x \sec^2 x \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$\frac{du}{\sec^2 x} = dx$

$$\int u^8 \cancel{\sec^2 x} \cdot \frac{du}{\cancel{\sec^2 x}}$$

$$= \frac{u^9}{9} + C$$

$$= \frac{1}{9} \tan^9 x + C$$

$$\int x^6 \sin(x^7)\,dx \qquad u = x^7$$

$$du = 7x^6\,dx$$

$$\int \cancel{x^6} \sin u \cdot \frac{du}{7\cancel{x^6}} \qquad \frac{du}{7x^6} = dx$$

$$= -\frac{1}{7}\cos u + C$$

$$= -\frac{1}{7}\cos(x^7) + C$$

$$\int \frac{1}{y^2} \sec\left(\frac{1}{y}\right) \tan\left(\frac{1}{y}\right) dy$$

$$u = \frac{1}{y} \qquad y^{-1}$$

$$du = \frac{-1}{y^2}\, dy \qquad -y^{-2}$$

$$-y^2\, du = dy$$

$$\int \frac{1}{\cancel{y^2}} \sec u \tan u \cdot -\cancel{y^2}\, du$$

$$= -\sec u + C$$

$$= -\sec\left(\frac{1}{y}\right) + C$$

$$\int \frac{8\cos(4x-7)}{\sin^6(4x-7)}\,dx$$

$$u = \sin(4x-7)$$

$$du = \cos(4x-7)\cdot 4\,dx$$

$$\frac{du}{4\cos(4x-7)} = dx$$

$$8\int \frac{\cancel{\cos(4x-7)}}{u^6}\cdot\frac{du}{4\cancel{\cos(4x-7)}}$$

$$2\int u^{-6}\,du$$

$$= \frac{2u^{-5}}{-5} + C$$

$$= \frac{-2}{5u^5} + C$$

$$= \frac{-2}{5\sin^5(4x-7)} + C$$

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