

PHASE SHIFT + VERTICAL SHIFT

(Horizontal shift)

$$y = x^2$$

$$y = (x-5)^2 + 2$$

Right 5, up 2

$$y = \sin\left(x + \frac{\pi}{4}\right) - 3$$

Left $\frac{\pi}{4}$ down 3

$$y = \sin 2\left(x + \frac{\pi}{4}\right) - 3$$

$$y = \sin\left(2x + \frac{\pi}{2}\right) - 3$$

$$2x + \frac{\pi}{2} = 0$$

$$\frac{1}{2} \cdot 2x = -\frac{\pi}{2} \cdot \frac{1}{2}$$

$$x = -\frac{\pi}{4}$$

With + little sec x + csc x

$$y = a \sin(bx+c) + d$$

sin
cos

csc
sec

amp

|a|

NA

period

$\frac{2\pi}{b}$

$\frac{2\pi}{b}$

vertical shift

d

d

phase shift
(horiz. shift)

$b\left(x + \frac{c}{b}\right)$

$b\left(x + \frac{c}{b}\right)$

$-\frac{c}{b}$

$x = -\frac{c}{b}$

$$bx + c = 0$$

$$bx = -c$$

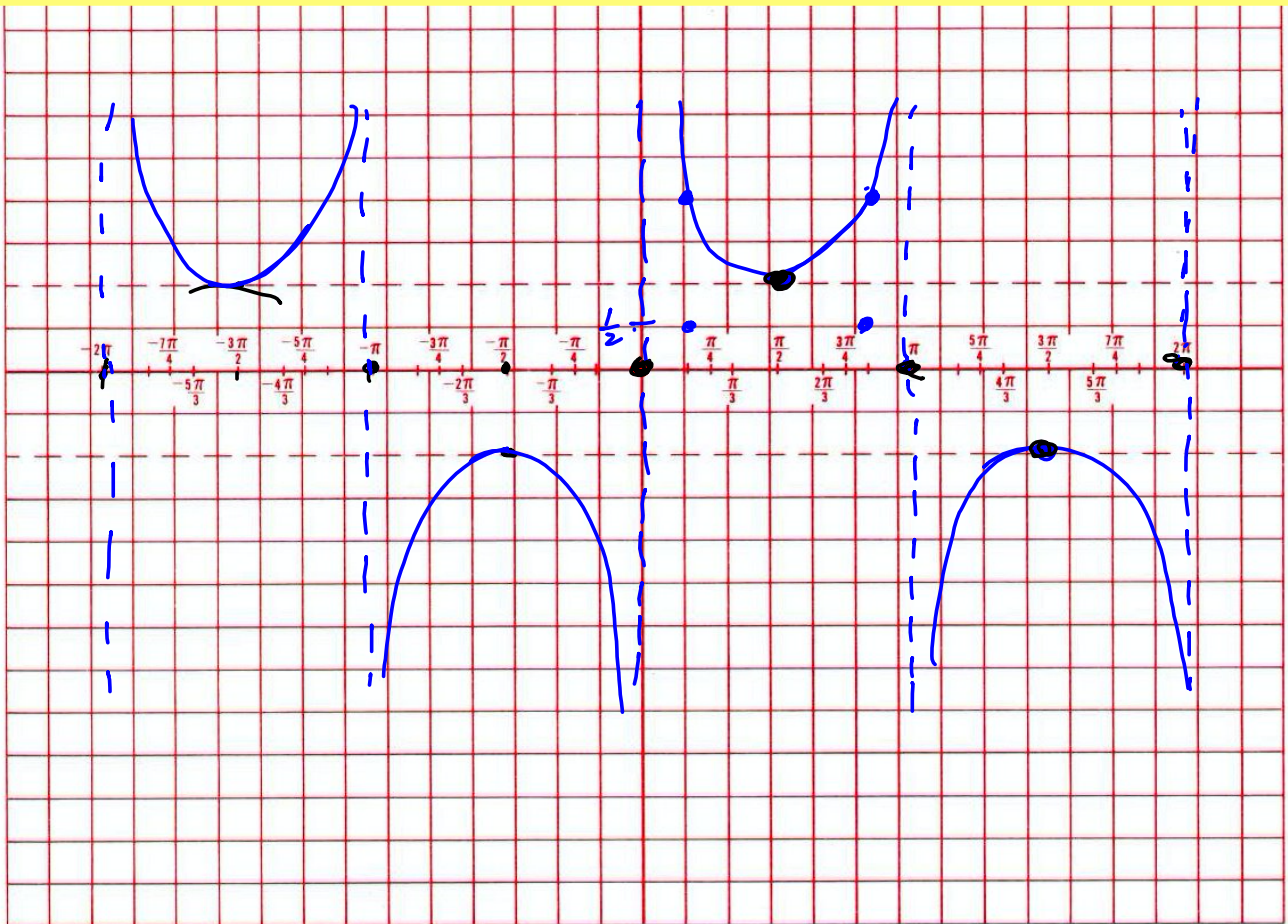
$$x = -\frac{c}{b}$$

How to find 5 points:

- 1) 1st point starts at phase shift.
- 2) Spacing: period $\cdot \frac{1}{4}$
- 3) Add spacing to phase shift to get 2nd pt.
(repeat for all pts.)
(Make common denominators if needed.)

$$y = \sin x$$

$$y = \csc x = \frac{1}{\sin x}$$

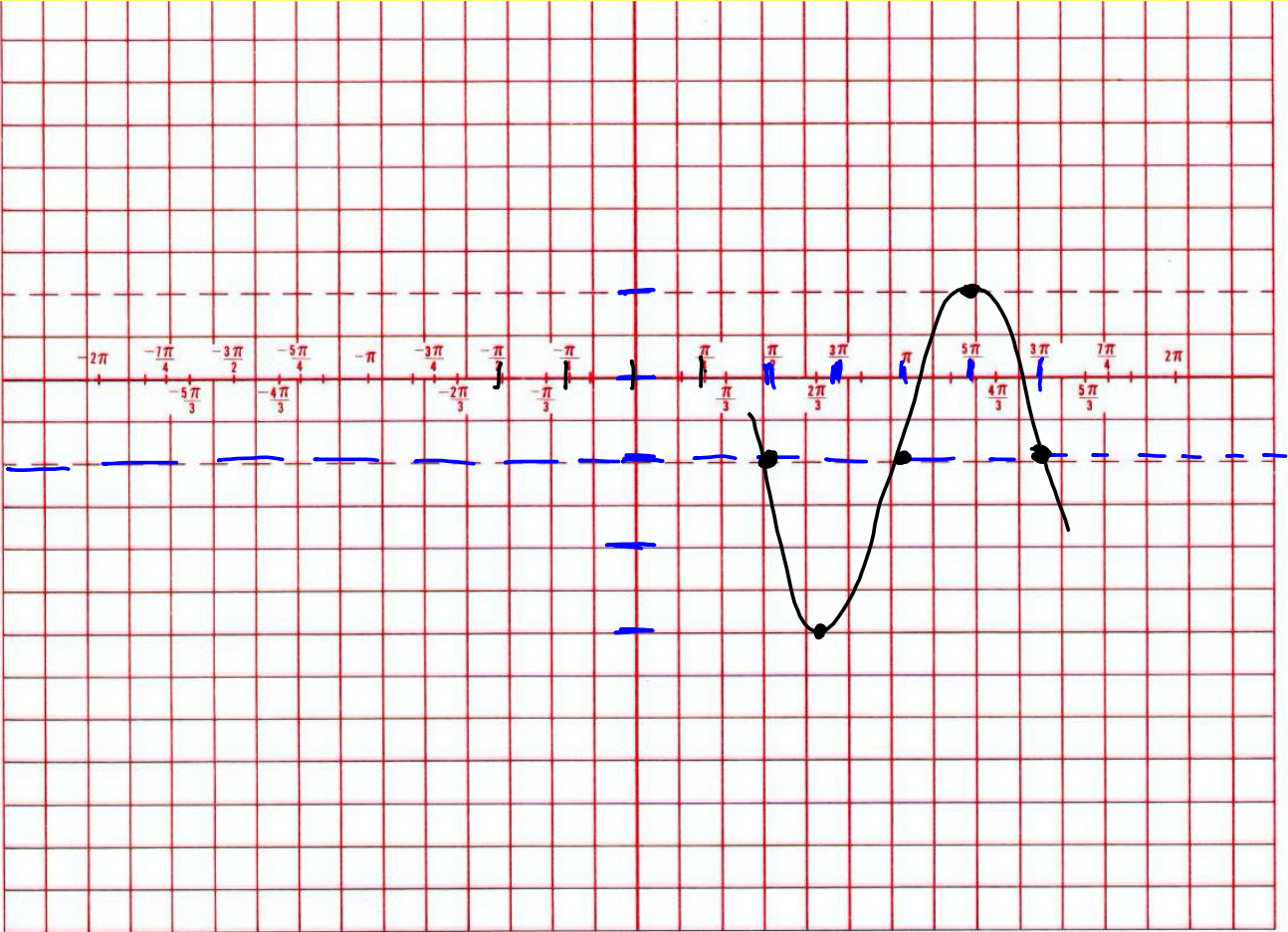


$$y = -2 \sin(2x - \pi) - 1$$

Amp 2(-) v.s. -1
 period $\frac{2\pi}{2}$ p.s. $2(x - \frac{\pi}{2})$
 $x = \pi/2$

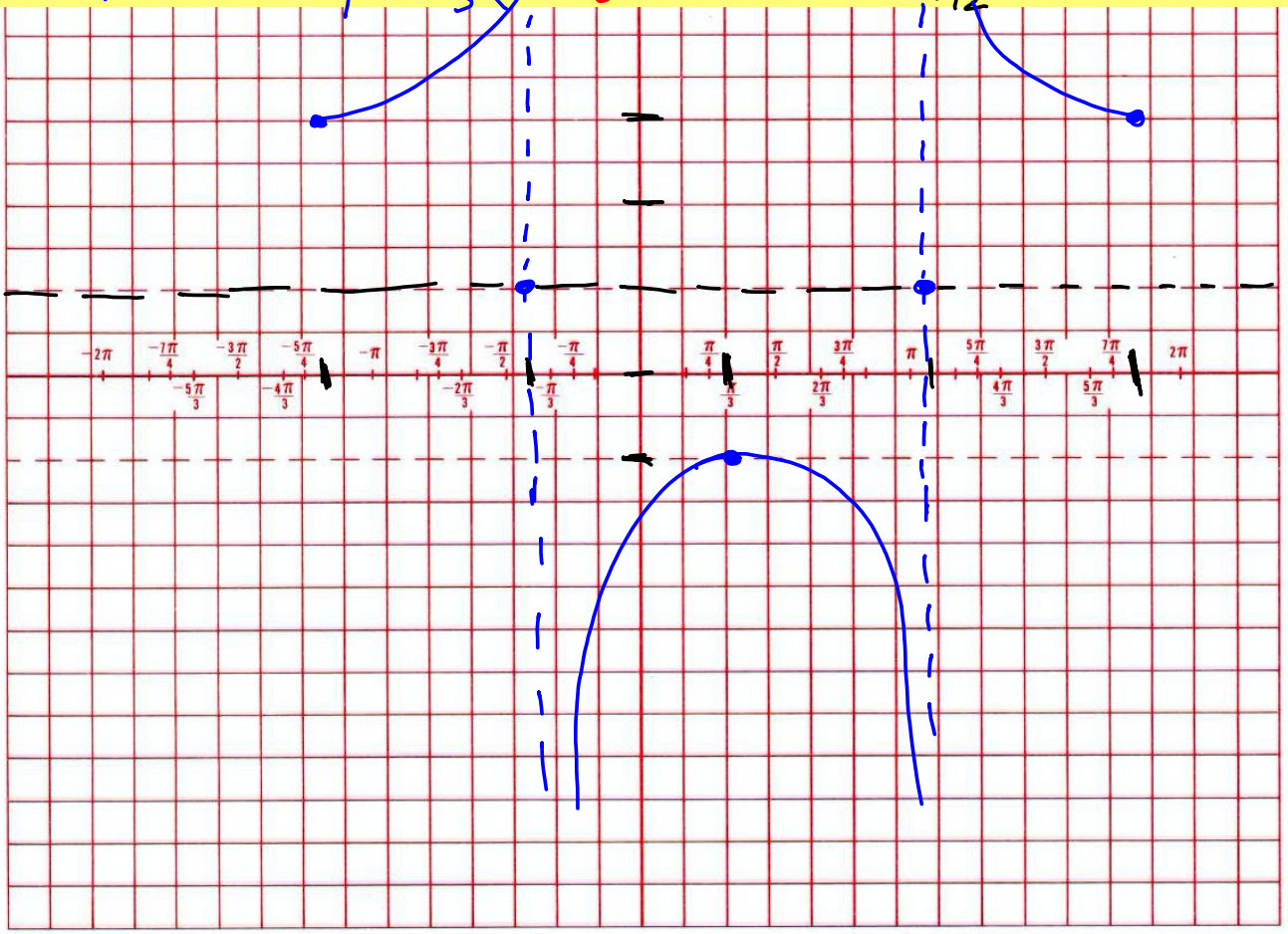
Spacing:
 per. $\frac{1}{4}$
 $\pi \cdot \frac{1}{4} = \frac{\pi}{4}$

$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$



$y = 1 + 2 \sec\left(\frac{2}{3}x + \frac{7\pi}{6}\right)$
 V.S. 1
 amp NA(2)
 cos
 period = $\frac{2\pi}{2/3} = 2\pi \cdot \frac{3}{2} = 3\pi$
 p.s. $\frac{2}{3}\left(x + \frac{7\pi}{6}\right)$ $x = -\frac{7\pi}{6}$
 spacing: $3\pi \cdot \frac{1}{4} = \frac{3\pi}{4}$

$-\frac{7\pi}{6}$	$-\frac{5\pi}{12}$	$\frac{\pi}{3}$	$\frac{13\pi}{12}$	$\frac{11\pi}{6}$
$-\frac{14\pi}{12}$	$-\frac{5\pi}{12}$	$\frac{4\pi}{12}$	$\frac{13\pi}{12}$	$\frac{22\pi}{12}$
$-\frac{12}{12}$				
$+\frac{9\pi}{12}$				

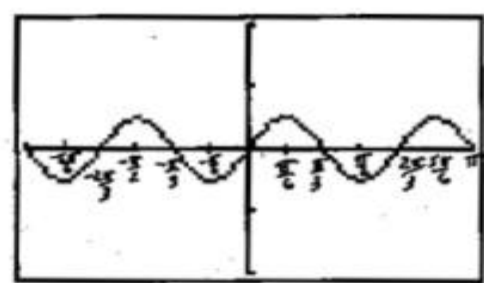


v.s. 1
 amp 1
 period π
 R-L 2 peaks
 $\frac{3\pi}{4} + \frac{\pi}{4} = \pi$

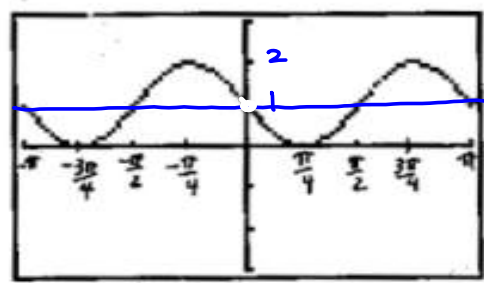
p.s. $-\frac{\pi}{4}$

$$y = 1 \cos 2(x + \frac{\pi}{4}) + 1$$

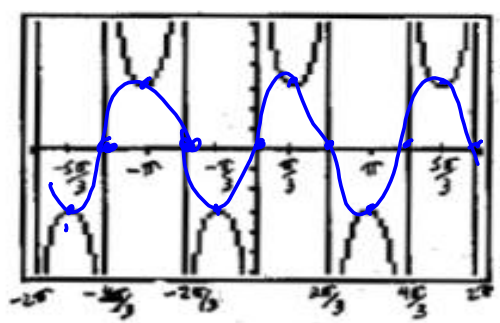
$$b = \frac{2\pi}{\text{period}} \quad \frac{2\pi}{\pi} = 2$$



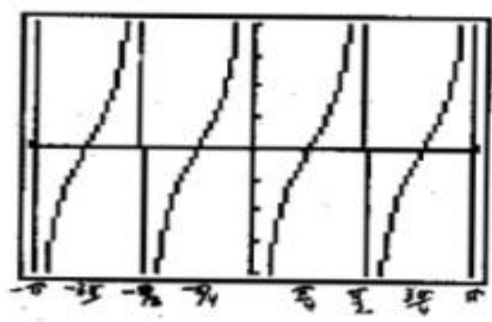
sin



cos



sec



tan