

CHAPTER 4 GRAPHS OF THE CIRCULAR FUNCTIONS

Connections (page 141)

1. One example is $f(x) = x^2$. 2. One example is $f(x) = x^3$. 3. (a) even (b) odd (c) neither

Connections (page 148)

1. $X = -.4161468, Y = .90929743$; X is $\cos 2$ and Y is $\sin 2$. 2. $X = 1.9, Y = .94630009$; $\sin 1.9 = .94630009$
 3. $X = 1.9, Y = -.3232896$; $\cos 1.9 = -.3232896$

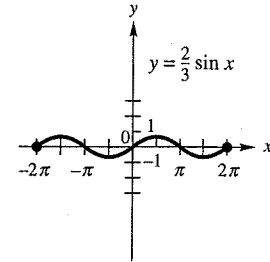
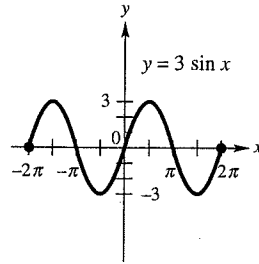
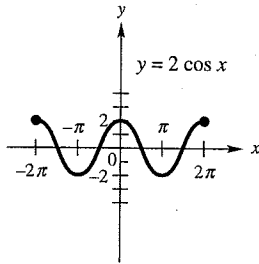
4.1 Exercises (page 150)

1. G 2. A 3. E 4. D 5. B 6. H 7. F 8. C

9. 2

10. 3

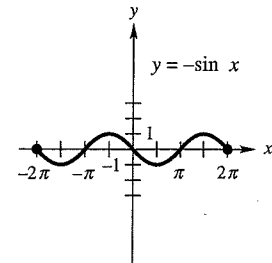
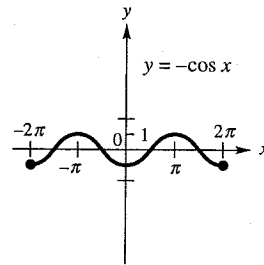
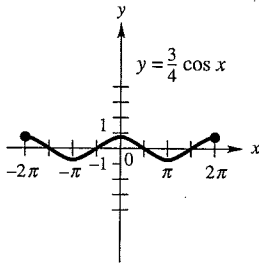
11. $\frac{2}{3}$



12. $\frac{3}{4}$

13. 1

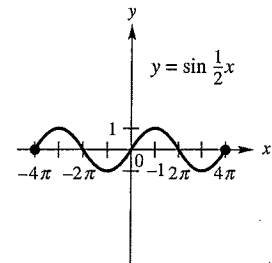
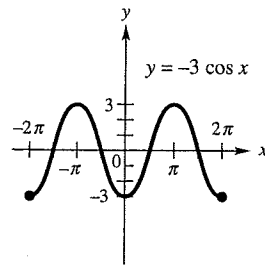
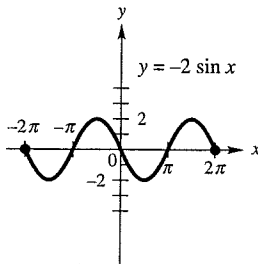
14. 1



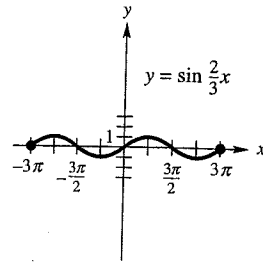
15. 2

16. 3

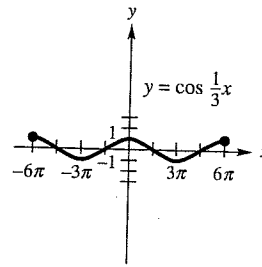
17. $4\pi; 1$



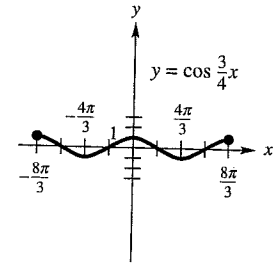
18. $3\pi; 1$



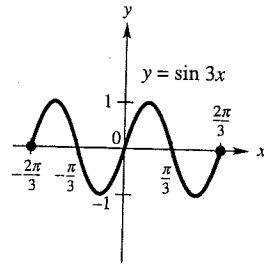
19. $6\pi; 1$



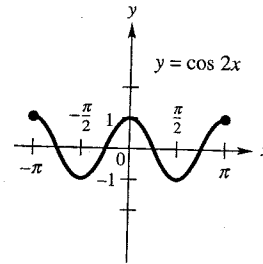
20. $\frac{8\pi}{3}; 1$



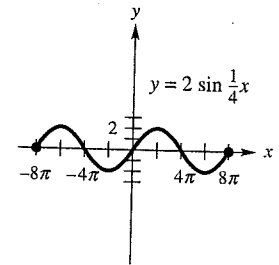
21. $\frac{2\pi}{3}; 1$



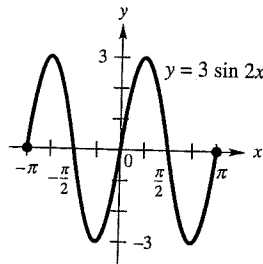
22. $\pi; 1$



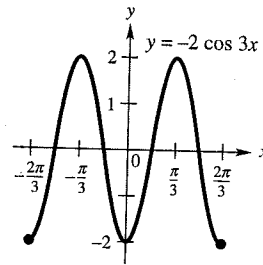
23. $8\pi; 2$



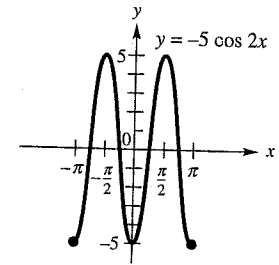
24. $\pi; 3$



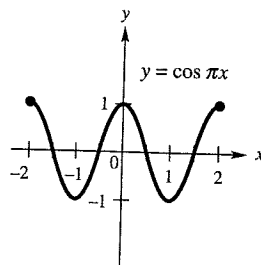
25. $\frac{2\pi}{3}; 2$



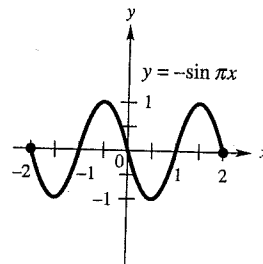
26. $\pi; 5$



27. $2; 1$



28. $2; 1$



There are other correct answers in Exercises 29–32.

29. $y = 4 \sin \frac{1}{2}x$ 30. $y = \frac{1}{4} \sin \frac{\pi}{2}x$

31. $y = 4 \cos \left(\frac{1}{2}x - \frac{\pi}{2} \right)$

32. $y = \frac{1}{4} \cos \left(\frac{\pi}{2}x - \frac{\pi}{2} \right)$

33. 24 hr

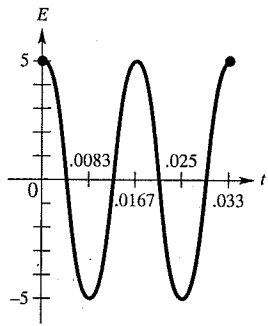
34. approximately $\frac{2.6 - .2}{2} = 1.2$ 35. approximately 6:00 P.M.; approximately .2 ft 36. approximately 7:19 P.M.; approximately 0 ft

37. approximately 2:00 A.M.; approximately 2.6 ft 38. approximately 3:18 A.M.; approximately 2.4 ft

39. (a) 20 (b) 75 40. (a) about 2 hr (b) 1 yr 41. (a) $80^\circ; 50^\circ$ (b) 15° (c) about 35,000 yr

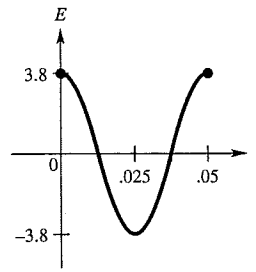
(d) downward 42. (a) $y = \frac{1}{3} \sin \frac{4\pi t}{3}$ (b) $\frac{3}{2}$ sec 43. 1; 240° or $\frac{4\pi}{3}$ 44. 1; 120° or $\frac{2\pi}{3}$

45. (a) $5; \frac{1}{60}$ (b) 60
 (c) 5; 1.545; -4.045; -4.045; 1.545
 (d)

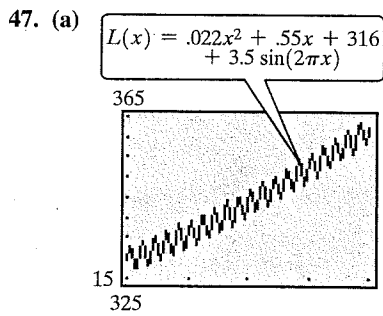


$E = 5 \cos 120\pi t$

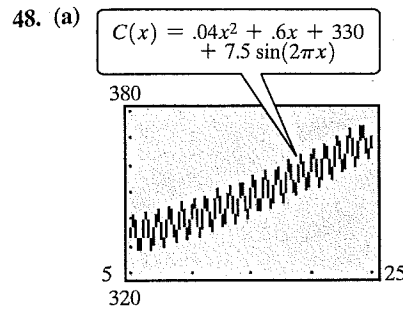
46. (a) $3.8; \frac{1}{20}$ (b) 20
 (c) -3.074; 1.174; -3.074; -3.074; 1.174
 (d)



$E = 3.8 \cos 40\pi t$



- (b) maximums: $x = \frac{1}{4}, \frac{5}{4}, \frac{9}{4}, \dots$;
 minimums: $x = \frac{3}{4}, \frac{7}{4}, \frac{11}{4}, \dots$



- (c) $C(x) = .04(x - 1970)^2 + .6(x - 1970) + 330 + 7.5 \sin[2\pi(x - 1970)]$

53. 1 54. 1 55. $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ 56. 0 and π 57. (a) 5 in. (b) 2 cycles per sec; $\frac{1}{2}$ sec (c) after $\frac{1}{4}$ sec

(d) approximately 4; After 1.3 seconds, the weight is about 4 inches above the equilibrium position. 58. (a) 4 in.

- (b) $\frac{5}{\pi}$ cycles per sec; $\frac{\pi}{5}$ sec (c) after $\frac{\pi}{10}$ sec (d) approximately 2; After 1.466 seconds, the weight is about 2 inches above

the equilibrium position. 59. (a) $y = -3 \cos 12t$ (b) $\frac{\pi}{6}$ sec 60. (a) $y = -2 \cos 6\pi t$ (b) 3 cycles per sec

4.2 Exercises (page 162)

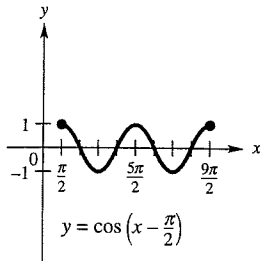
1. B 2. D 3. C 4. A 5. D 6. G 7. H 8. A 9. B 10. E 11. F 12. C

13. right 14. left 15. 2; 2π ; none; π to the right 16. $\frac{2}{3}$; 2π ; none; $\frac{\pi}{2}$ to the left 17. 4; 4π ; none; π to the left

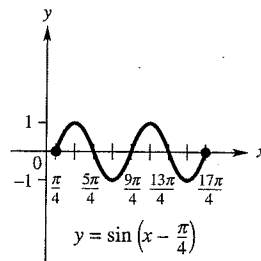
18. 1; 3π ; none; $\frac{\pi}{3}$ to the right 19. 3; π ; none; $\frac{\pi}{4}$ to the right 20. $\frac{1}{2}$; 4π ; none; 2π to the left

21. 1; $\frac{2\pi}{3}$; up 2; $\frac{\pi}{15}$ to the right 22. $\frac{1}{2}$; π ; down 1; $\frac{3\pi}{2}$ to the right

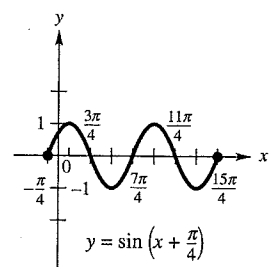
23.



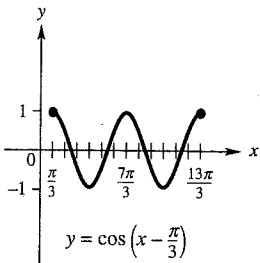
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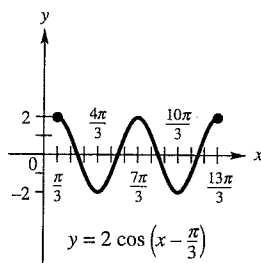
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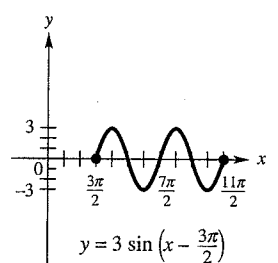
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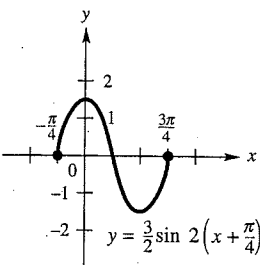
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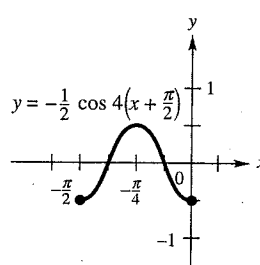
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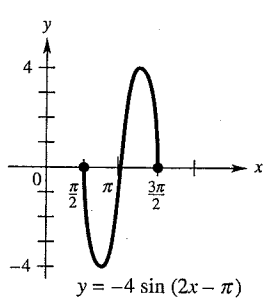
29.



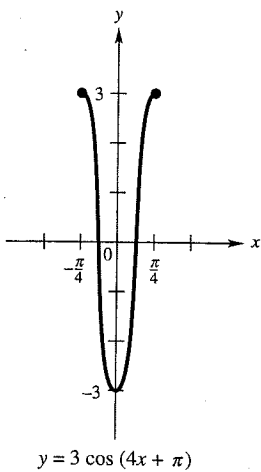
30.



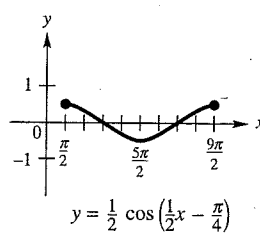
31.



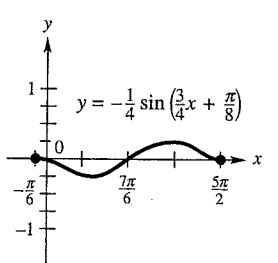
32.



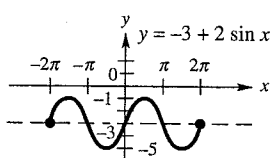
33.



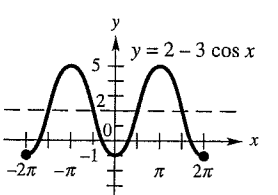
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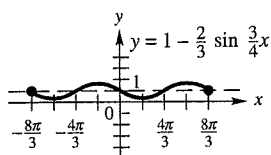
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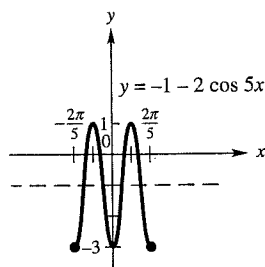
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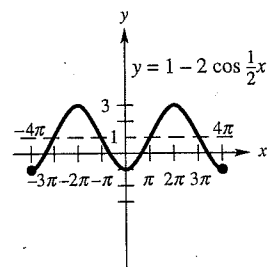
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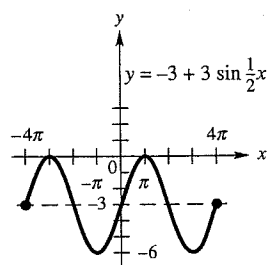
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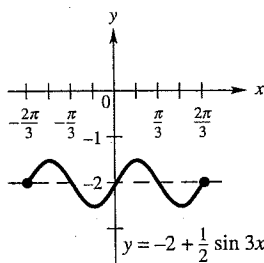
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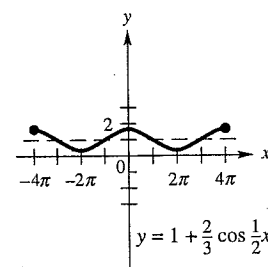
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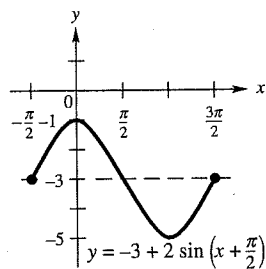
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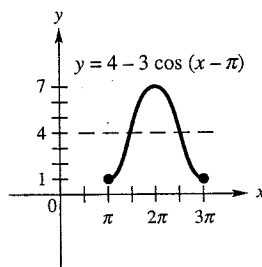
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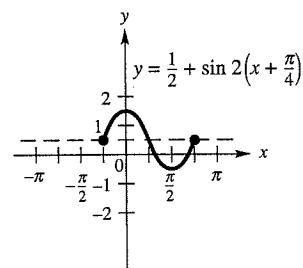
43.



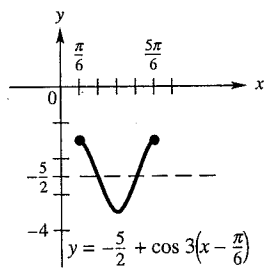
44.



45.



46.



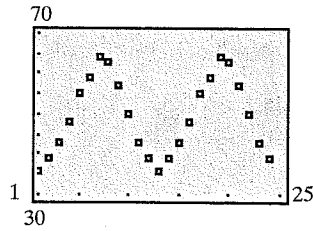
48. $a = 1, \alpha = \frac{\pi}{3}$

There are other correct answers in Exercises 49–52.

49. $y = 3 \sin 2(x - \frac{\pi}{4})$ 50. $y = \pi \sin \pi(x - .5)$

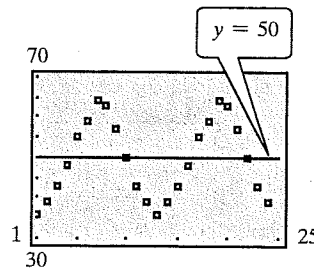
51. $y = 3 \cos 2(x - \frac{\pi}{2})$ 52. $y = \pi \cos \pi(x - 1)$

53. (a) yes



(b) It represents the average yearly temperature.

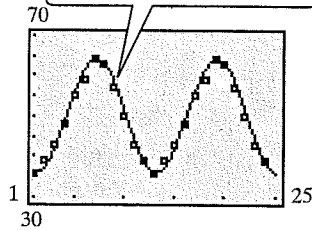
(c) 14; 12; 4.2



(d) $f(x) = 14 \sin\left[\frac{\pi}{6}(x - 4.2)\right] + 50$

(e) The function gives an excellent model for the given data.

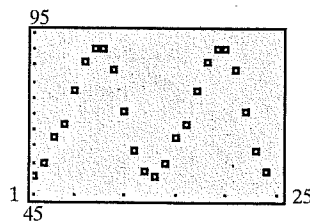
$f(x) = 14 \sin\left[\frac{\pi}{6}(x - 4.2)\right] + 50$



```
SinReg
y=a*sin(bx+c)+d
a=13.21
b=.52
c=-2.18
d=49.68
```

TI-83 fixed to the nearest hundredth

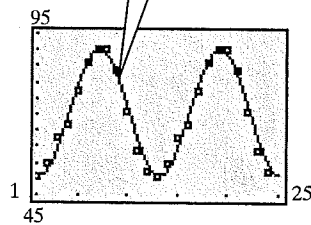
54. (a) 70.4° (b)



(c) $f(x) = 19.5 \cos\left[\frac{\pi}{6}(x - 7.2)\right] + 70.5$

(d) The function gives an excellent model for the data.

$f(x) = 19.5 \cos\left[\frac{\pi}{6}(x - 7.2)\right] + 70.5$



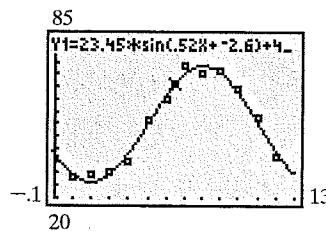
```
SinReg
y=a*sin(bx+c)+d
a=19.72
b=.52
c=-2.17
d=70.47
```

TI-83 fixed to the nearest hundredth

55. (a)

```
SinReg
y=a*sin(bx+c)+d
a=23.45
b=.52
c=-2.60
d=49.94
```

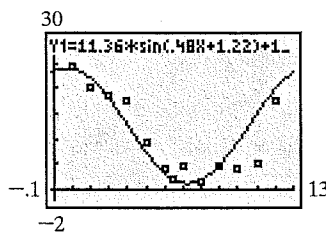
TI-83 fixed to the nearest hundredth



(b)

```
SinReg
y=a*sin(bx+c)+d
a=11.36
b=.48
c=1.22
d=12.69
```

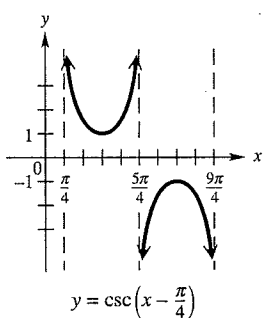
TI-83 fixed to the nearest hundredth



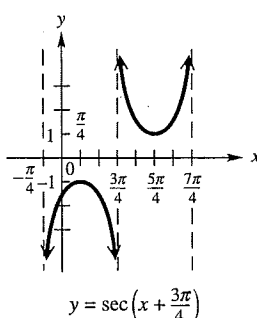
4.3 Exercises (page 177)

1. true 2. false; The smallest such k is π . 3. true 4. false; Secant values are undefined when $x = \frac{\pi}{2} + n\pi$, while cosecant values are undefined when $x = n\pi$. 5. false; $\tan(-x) = -\tan x$ for all x in the domain. 6. true
 7. B 8. C 9. E 10. A 11. D 12. F

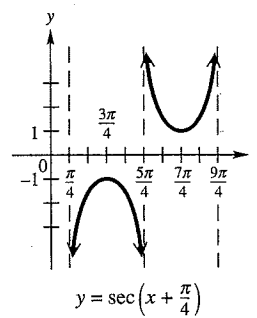
13.



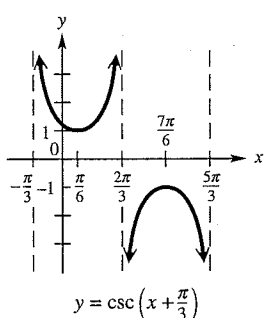
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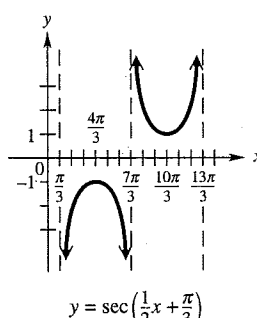
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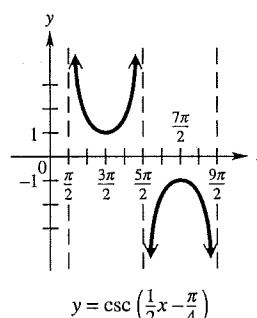
16.



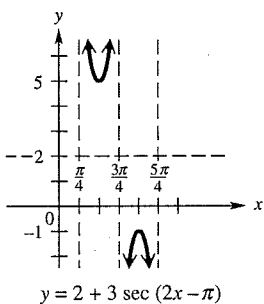
17.



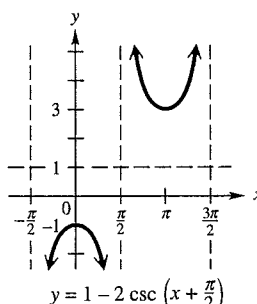
18.



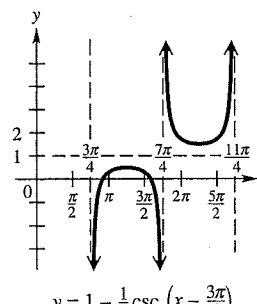
19.



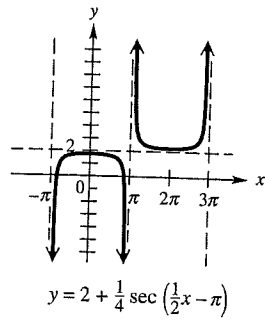
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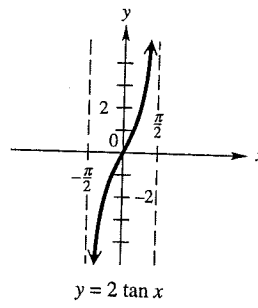
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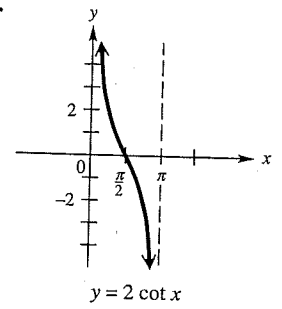
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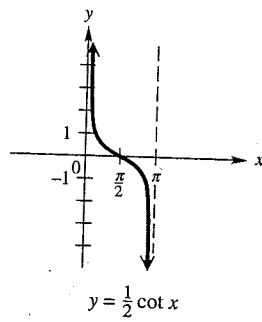
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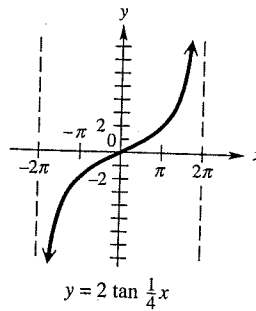
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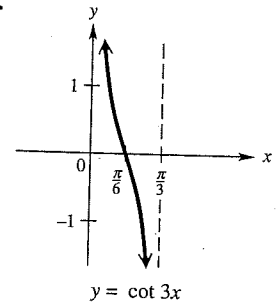
25.



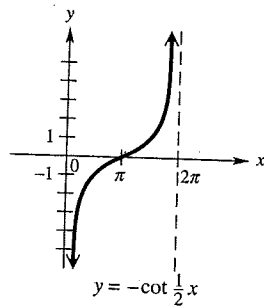
26.



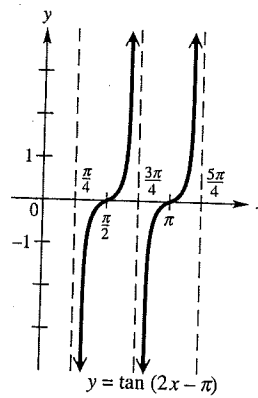
27.



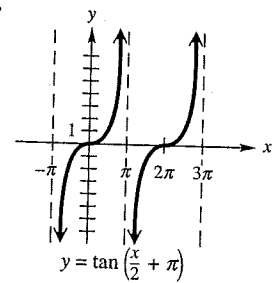
28.



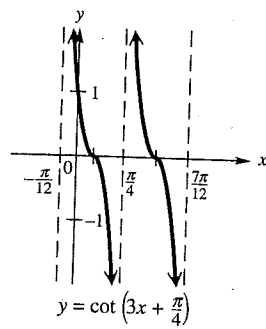
29.



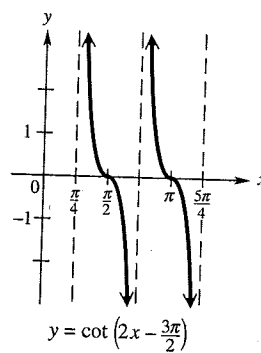
30.



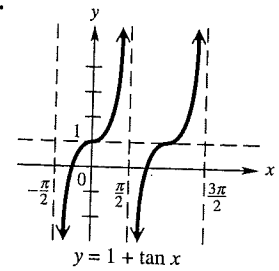
31.



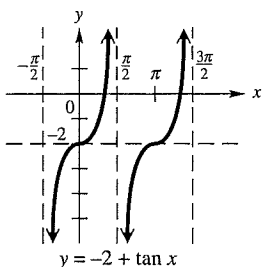
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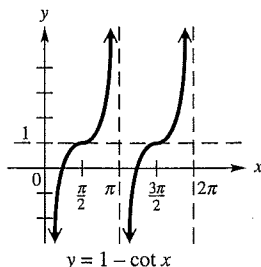
33.



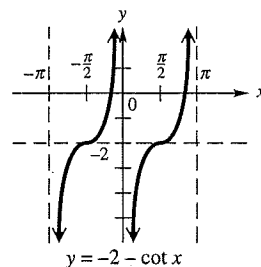
34.



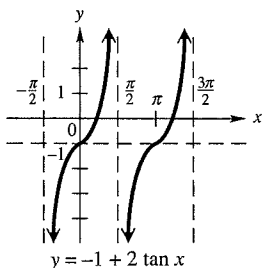
35.



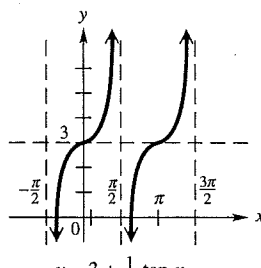
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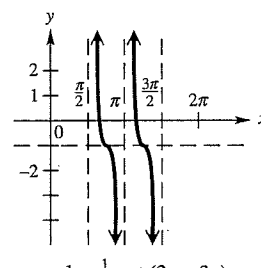
37.



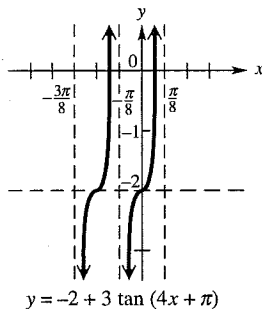
38.



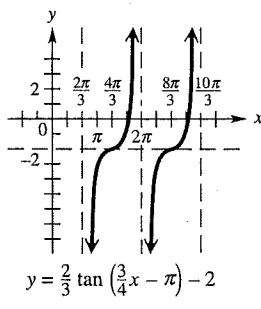
39.



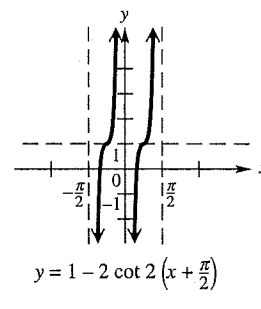
40.



41.



42.



43. domain: $\{x \mid x \neq (2n + 1)\frac{\pi}{4}, \text{ where } n \text{ is an integer}\}$; range: $(-\infty, \infty)$

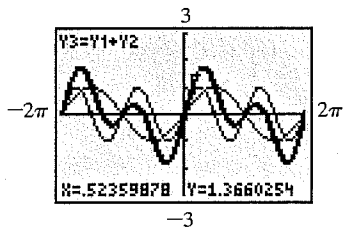
44. domain: $\{x \mid x \neq \frac{n\pi}{4}, \text{ where } n \text{ is an integer}\}$; range: $(-\infty, -2] \cup [2, \infty)$ 45. four 46. none

47. (a) 0 m (b) -2.9 m (c) -12.3 m (d) 12.3 m (e) It leads to $\tan \frac{\pi}{2}$, which is undefined.

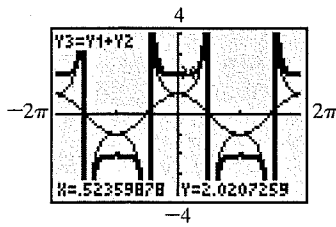
48. (a) 4 m (b) 6.3 m (c) 63.7 m

In Exercises 51 and 52, we show the display for $Y_1 + Y_2$ at $x = \frac{\pi}{6}$.

51.



52.



53. π

54. $\frac{5\pi}{4}$

55. $y = \frac{5\pi}{4} + n\pi$

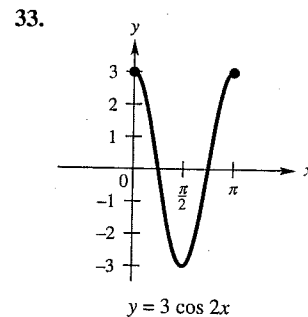
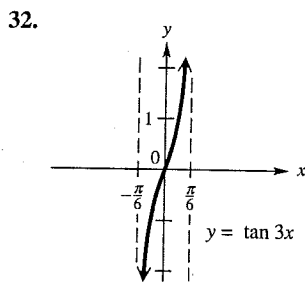
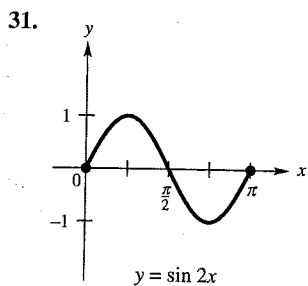
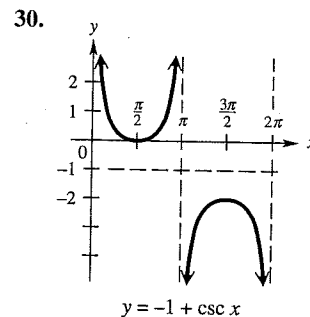
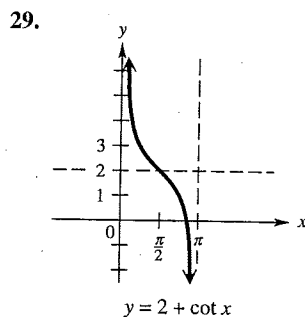
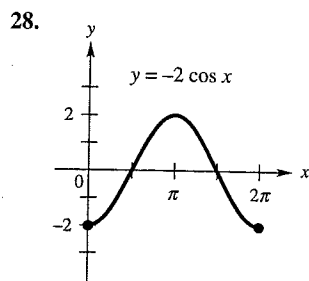
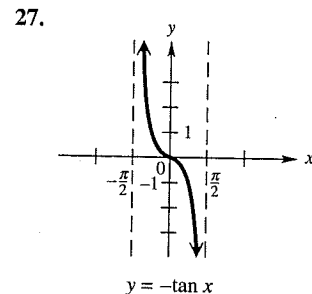
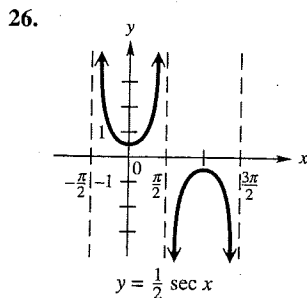
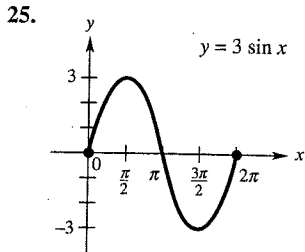
56. approximately .3217505544

57. approximately 3.463343208

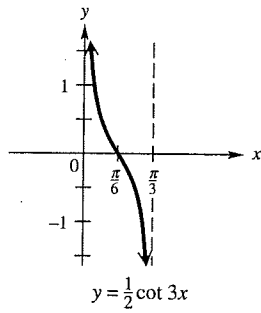
58. $.3217505544 + n\pi$

Chapter 4 Review Exercises (page 180)

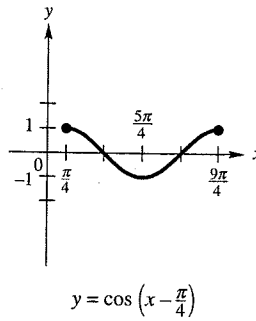
1. B 2. D 3. sine, cosine, tangent, and cotangent 4. secant, cosecant, tangent, and cotangent
 5. $2; 2\pi; \text{none}; \text{none}$ 6. not applicable; $\frac{\pi}{3}; \text{none}; \text{none}$ 7. $\frac{1}{2}; \frac{2\pi}{3}; \text{none}; \text{none}$ 8. $2; \frac{2\pi}{5}; \text{none}; \text{none}$
 9. $2; 8\pi; 1 \text{ up}; \text{none}$ 10. $\frac{1}{4}; 3\pi; 3 \text{ up}; \text{none}$ 11. $3; 2\pi; \text{none}; \frac{\pi}{2} \text{ to the left}$ 12. $1; 2\pi; \text{none}; \frac{3\pi}{4} \text{ to the right}$
 13. not applicable; $\pi; \text{none}; \frac{\pi}{8} \text{ to the right}$ 14. not applicable; $2; \text{none}; 2 \text{ to the right}$
 15. not applicable; $\frac{\pi}{3}; \text{none}; \frac{\pi}{9} \text{ to the right}$ 16. not applicable; $2\pi; \text{none}; \frac{3\pi}{2} \text{ to the left}$ 17. tangent 18. sine
 19. cosine 20. cosecant 21. cotangent 22. secant



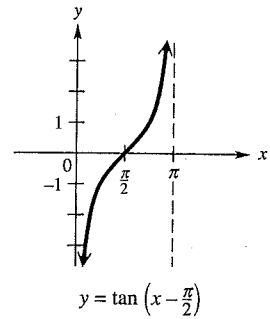
34.



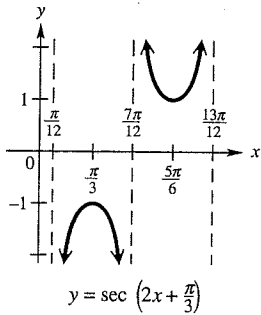
35.



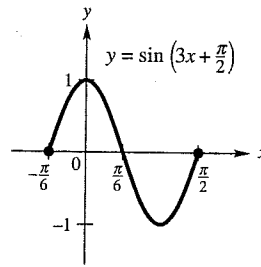
36.



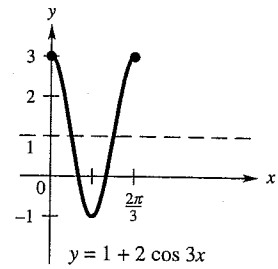
37.



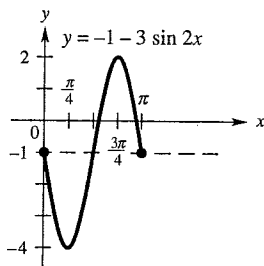
38.



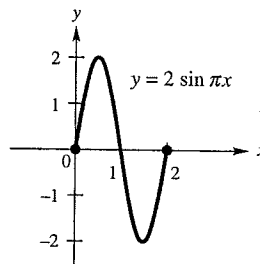
39.



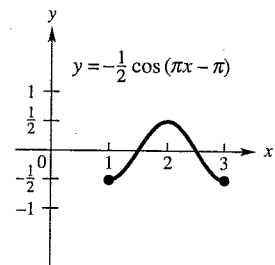
40.



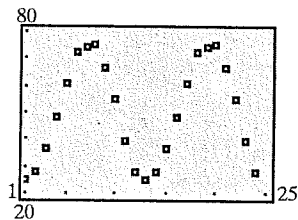
41.



42.



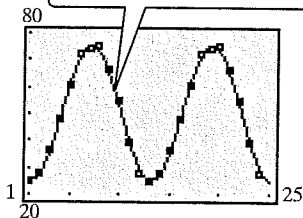
45. (a)



(b) $f(x) = 25 \sin\left[\frac{\pi}{6}(x - 4.2)\right] + 50$

(d) The function gives an excellent model for the data.

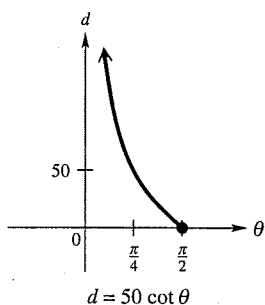
$f(x) = 25 \sin\left[\frac{\pi}{6}(x - 4.2)\right] + 50$



(e)

```
SinReg
y=a*sin(bx+c)+d
a=25.77
b=.52
c=-2.19
d=50.57
```

46. (b)



47. (a) 100 (b) 258 (c) 122 (d) 296

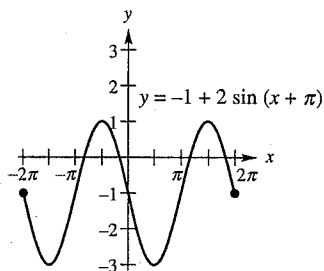
48. (a) about 20 yr (b) maximum: about 150,000; minimum: about 5000

50. 1; no

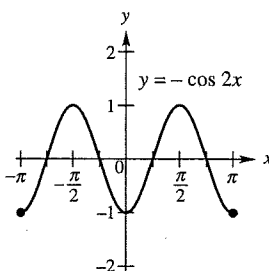
Chapter 4 Test (page 182)

1. (a) π (b) 6 (c) $[-3, 9]$ (d) -3 (e) $\frac{\pi}{4}$ to the left (that is, $-\frac{\pi}{4}$)

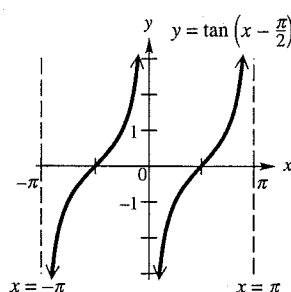
2.



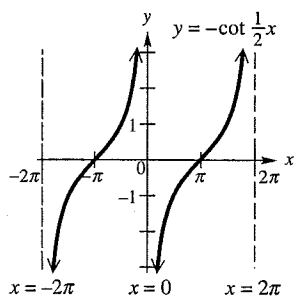
3.



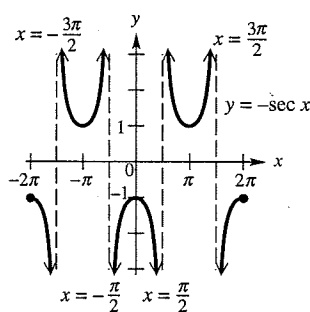
4.



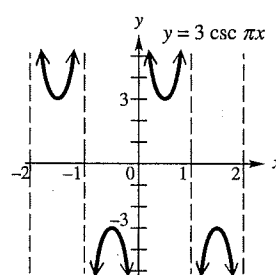
5.



6.

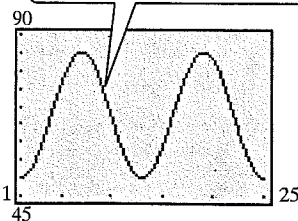


7.



8. (a)

$$f(x) = 17.5 \sin\left[\frac{\pi}{6}(x - 4)\right] + 67.5$$



(b) 17.5; 12; 4 to the right; 67.5 up

(c) approximately 52°F

(d) 50°F in January; 85°F in July

(e) approximately 67.5°; This is the vertical translation.

10. C

CHAPTER 5 TRIGONOMETRIC IDENTITIES

5.1 Exercises (page 191)

1. -2.6 2. $-.65$ 3. $.625$ 4. $-.75$ 5. $\frac{\sqrt{7}}{4}$ 6. $-\frac{3\sqrt{10}}{10}$ 7. $-\frac{2\sqrt{5}}{5}$ 8. $-\frac{\sqrt{77}}{11}$