More Chain Rule

$f(x) = \cos(3x^2 - 7x)$

$f(x) = -\sin(3x^2 - 7x) \cdot (6x - 7)$

$f(x) = \tan^8(x^5 - 3x^4) = [\tan(x^5 - 3x^4)]^8$

$f'(x) = 8\tan^7(x^5 - 3x^4) \cdot \sec^2(x^5 - 3x^4) \cdot (5x^4 - 12x^3)$

$f(x) = \csc^5(\cot(3x^7))$ $\quad \underbrace{\csc^5 x}\,\underbrace{(\cot(3x^7))}$

$f'(x) = 5\csc^4(\cot(3x^7)) \cdot -\underline{\csc}(\cot(3x^7))\underline{\cot}(\cot(3x^7)) \cdot -\csc^2(3x^7) \cdot 21x^6$

$f(x) = [\csc^5 x][\cot(3x^7)]$

$f'(x) = \underbrace{\csc^5 x} \cdot \underbrace{-\csc^2(3x^7) \cdot 21x^6} + \underbrace{\cot(3x^7)} \cdot \underbrace{5\csc^4 x \cdot -\csc x \cot x}$

$$f(x) = \tan\left(\sec(x^4-2x)^6\right)$$

$$f'(x) = \underbrace{\sec^2\left(\sec(x^4-2x)^6\right)} \cdot \underbrace{\sec(x^4-2x)^6 \tan(x^4-2x)^6}$$

$$\cdot \underbrace{6(x^4-2x)^5} \cdot \underbrace{(4x^3-2)}$$

DIFFERENTIALS

$y = f(x)$

$\frac{dy}{dx} = f'(x)$

differentials

$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

Find dy.

$y = x^3 - 3x^2 + 7$

$\cancel{dx} \cdot \frac{dy}{\cancel{dx}} = 3x^2 - 6x$

$dy = (3x^2 - 6x)\,dx$

The radius of a sphere is measured to be 20 in. with a possible of ± 0.3 in.

Estimate the possible error in the volume.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$dV = 4\pi r^2 dr$$

$$dV = 4\pi (20)^2 \cdot \pm(0.3) = \pm 150.8 \text{ in}^3$$

% error radius

$$\frac{\text{actual} - \text{theor}}{\text{actual}} = \frac{dr}{r} = \frac{\pm 0.3}{20} = 0.045$$

4.5%

% error of Volume

$$\frac{dV}{V} = \frac{\cancel{4\pi r^2} dr}{\frac{\cancel{4}}{3}\cancel{\pi} r^3} \cdot \frac{3}{\cancel{4}} = 3 \cdot \frac{dr}{r} = 3 \cdot 4.5\% = 13.5\%$$

Dome of a silo – 12 ft radius

Hemisphere

Coat of paint 0.002 ft thick

Estimate the volume of the coating of paint.

dV

$V = \frac{2}{3}\pi r^3$

$dV = 2\pi r^2\, dr$

$= 2\pi (12)^2 (0.002) = 1.81 \text{ ft}^3$