Radical Equations

$$\sqrt{3x+4} - \sqrt{x+2} = 2$$

$$1^2 + 2^2 = 3^2$$
$$1 + 4 \neq 9$$

$$\left(\sqrt{3x+4}\right)^2 = \left(2 + \sqrt{x+2}\right)^2$$
$$(2+\sqrt{x+2})(2+\sqrt{x+2})$$

$$3x+4 = 4 + 2\sqrt{x+2} + 2\sqrt{x+2} + x+2$$

$$3x+4 = 4\sqrt{x+2} + x + 6$$
$$-x-6$$

$$\left(2x-2\right)^2 = \left(4\sqrt{x+2}\right)^2$$
$$(2x-2)(2x-2)$$

$$x = -1$$
$$\sqrt{1} - \sqrt{1} = 2$$
$$1 - 1 = 2$$

$$x = 7$$
$$\sqrt{25} - \sqrt{9} = 2$$
$$5 - 3 = 2 \checkmark$$

$$4x^2 - 8x + 4 = 16(x+2)$$

$$4x^2 - 8x + 4 = 16x + 32$$
$$-16x - 32$$

$$4x^2 - 24x - 28 = 0$$

$$4(x^2 - 6x - 7) = 0$$

$$4(x+1)(x-7) = 0$$

$$x = -1, 7$$

extraneous solution

PARTIAL FRACTIONS — to find the original fractions that were combined to make the given fraction

$$\frac{(x-4)}{(x-4)}\frac{2}{x+3} + \frac{6}{x-4}\frac{(x+3)}{(x+3)} = \frac{2x-8+6x+18}{(x+3)(x-4)} = \frac{8x+10}{(x+3)(x-4)}$$

$(x+3)$
$(x-4)$ $\quad \dfrac{8x+10}{(x+3)(x-4)} = \dfrac{A(x+3)(x-4)}{x+3} + \dfrac{B(x+3)(x-4)}{x-4}$

$$\frac{Ax' + B}{x^2 + 5}$$

$$8x+10 = A(x-4) + B(x+3)$$

$$8x+10 = Ax - 4A + Bx + 3B$$

$$4\left[8 = A+B\right]$$
$$10 = -4A+3B$$

$$32 = 4A + 4B$$
$$10 = -4A + 3B$$
$$42 = 7B$$
$$6 = B$$

$$8 = A + 6$$
$$2 = A$$

$$\boxed{\frac{2}{x+3} + \frac{6}{x-4}}$$

Matrix Eq.

$$\begin{bmatrix} 1 & 1 \\ -4 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

A B

$$\frac{}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$$

$$\frac{}{(x^2+4)(x^3+7)} = \frac{Ax+B}{x^2+4} + \frac{Cx^2+Dx+E}{x^3+7}$$

$$\frac{}{x^3(4x+1)} = \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{4x+1}$$
$$\frac{(x)^3 \quad (x)^2}{}$$

$$\frac{}{(x-5)^2(x+3)} = \frac{A}{(x-5)^2} + \frac{B}{x-5} + \frac{C}{x+3}$$

$$\frac{10x^2 + 24x + 8}{(x^3 + 3x^2 + 4x + 12)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4}$$

$$x^2(x+3) + 4(x+3)$$
$$(x+3)(x^2+4)$$

1) Factor denom

2) Set up fractions

3) Multiply by common denom (original denom)

$$\frac{(x+3)}{(x^2+4)} \frac{10x^2 + 24x + 8}{(x+3)(x^2+4)} = \frac{A}{x+3} \frac{(x+3)}{(x^2+4)} + \frac{Bx+C}{x^2+4} (x+3)(x^2+4)$$

4) Set like coeff. =

$$10x^2 + 24x + 8 = A(x^2+4) + (Bx+c)(x+3)$$

$$10x^2 + 24x + 8 = Ax^2 + 4A + Bx^2 + 3Bx + Cx + 3C$$

$$10 = A + B$$
$$24 = \qquad 3B + C$$
$$8 = 4A \qquad + 3C$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \\ 4 & 0 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 10 \\ 24 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 0 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

$$\frac{5x+3}{x^2-1} + \frac{-4}{x+2}$$

$$\boxed{\frac{2}{x+3} + \frac{8x}{x^2+4}}$$