September 10, 2021

LIMITS

- $x \to \#$
  - sub # in
    - $\frac{0}{0}$
      - factor
      - conjugate
      - common denom
    - $\#0$
      - check one-sided limits for $\pm\infty$ OR DNE
- $x \to \pm\infty$
  - fraction
    - use highest power terms from num & denom
  - polyn
    - use highest power term
- Special Functions
  - Visualize the graph

$$\lim_{x \to \pi/6} \frac{\sin x}{2} = \frac{\sin \pi/6}{2} = \frac{1/2}{2} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\lim_{x \to 0} \frac{\sin nx}{nx} = 1$$

$$\lim_{x \to 0} \frac{5 \cdot \sin(5x)}{5 \cdot x} = 5 \lim_{x \to 0} \frac{\sin(5x)}{5x}$$

$$= 5 \cdot 1$$

$$= 5$$

$$\lim_{x \to 0} \frac{\sin x}{3x}$$

$$= \frac{1}{3} \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \frac{1}{3} \cdot 1 = \frac{1}{3}$$

$$\lim_{x \to 0} \frac{\frac{1}{x} \sin 6x}{\frac{1}{x} \sin 8x}$$

$$\lim_{x \to 0} \frac{\frac{6}{6} \cdot \frac{\sin 6x}{x}}{\frac{8}{8} \frac{\sin 8x}{x}}$$

$$\frac{6}{8} \lim_{x \to 0} \frac{\frac{\sin 6x}{6x}}{\frac{\sin 8x}{8x}}$$

$$\frac{3}{4} \cdot \frac{1}{1} = \frac{3}{4}$$

$$\lim_{x\to 0} \frac{1-\cos nx}{nx} = 0$$

$$\lim_{x\to 0} \frac{3(1-\cos 12x)}{3\cdot 4x}$$

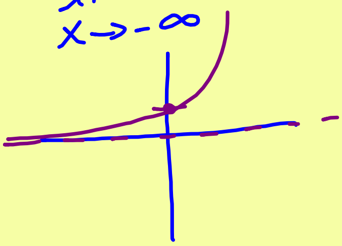
$$3\lim_{x\to 0} \frac{1-\cos 12x}{12x}$$

$$= 3 \cdot 0$$

$$= 0$$

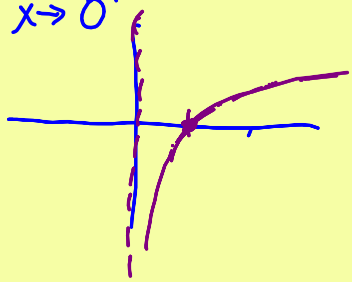
$$\lim_{x\to\infty} e^x = +\infty$$

$$\lim_{x\to-\infty} e^x = 0$$



$$\lim_{x\to\infty} \ln x = +\infty$$

$$\lim_{x\to 0^+} \ln x = -\infty$$



$$\lim_{x\to 0^+} e^{1/x} = e^{\infty} = \infty$$

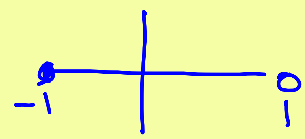
$$\lim_{x\to 0^+} \frac{1}{x} = \frac{+}{+} = +\infty$$

$$\lim_{x\to-\infty} \frac{1-e^x}{e^{-x}} = \frac{1-0}{\infty} = \frac{1}{\infty} = 0$$

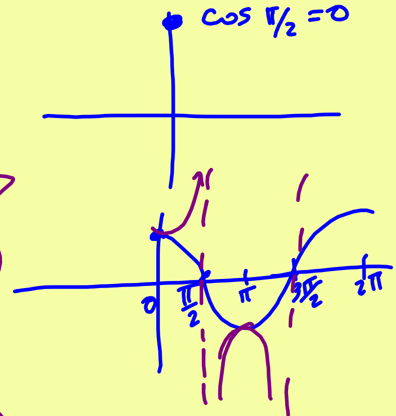
$$e^{+\,+\infty}$$

$$\lim_{x\to\pi/4} \ln(1-\tan x) = \ln\left(1-\tan\frac{\pi}{4}\right) = \ln(1-1) = \ln 0 = -\infty$$

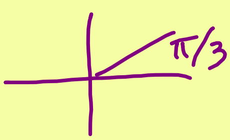
$$\lim_{x\to\pi^+} \sec x = \sec \pi$$
$$= -1$$



$$\lim_{x\to\frac{\pi}{2}^-} \sec x = +\infty$$



$$\lim_{x\to \frac{1}{2}^+} \frac{\cos^{-1} x}{\sin 2\pi x} = \frac{\pi/3}{\sin \cancel{2}\pi\cdot\frac{1}{\cancel{2}}}$$



$$= \frac{\frac{\pi}{3}}{0}$$
$$= \frac{+}{-}$$
$$= -\infty$$