DERIVATIVES OF $\ln x$, $\log_b x$, e^x, a^x, + x^x

$f(x)=e^x$ $\quad \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$x^2 \cdot x^3 = x^5$

$\lim_{h\to 0} \frac{e^{x+h}-e^x}{h}$

$\frac{d}{dx} e^x = e^x$

$\lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h}$

$\lim_{h\to 0} e^x \frac{(e^h-1)}{h} = e^x \cdot 1$

$f(x) = e^{x^2+4x}$

$f'(x) = e^{x^2+4x} \cdot (2x+4)$

$f(x) = x^2 \cdot e^{4x^3}$

$f'(x) = \underbrace{x^2 \cdot e^{4x^3} \cdot 12x^2} + \underbrace{e^{4x^3} \cdot 2x}$

$= 2x e^{4x^3}[6x^3 + 1]$

Power

x^n

Find $\frac{dy}{dx}$

$y = a^x$

$\ln y = \ln a^x$

$\ln y = x \cdot \ln a$

$\frac{1}{y}\frac{dy}{dx} = \ln a$

$\frac{dy}{dx} = y \cdot \ln a$

$= a^x \cdot \ln a$

$$\boxed{\frac{d}{dx} a^x = \ln a \cdot a^x}$$

$y = \ln x$

$e^y = e^{\ln x}$

$e^y = x$

$e^y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{e^{\ln x}} = \frac{1}{x}$

$$\boxed{\frac{d}{dx} \ln x = \frac{1}{x}}$$

$f(x) = 5^{x^2}$

$f'(x) = \ln 5 \cdot 5^{x^2} \cdot 2x$

$f(x) = 14^{\cos x}$

$f'(x) = \ln 14 \cdot 14^{\cos x} \cdot -\sin x$

$= -\ln 14 \cdot 14^{\cos x} \sin x$

$f(x) = e^{2x^7}$

$= \cancel{\ln e} \cdot e^{2x^7} \cdot 14x^6$

$f(x) = 7^{\frac{e^{x^3}}{\tan x}}$

$f'(x) = \ln 7 \cdot 7^{\frac{e^{x^3}}{\tan x}} \cdot \left[\dfrac{\tan x \cdot e^{x^3} \cdot 3x^2 - e^{x^3} \cdot \sec^2 x}{\tan^2 x}\right]$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$f(x) = \ln(x^3 - 4x^5)$$

$$f'(x) = \frac{1}{x^3 - 4x^5}\cdot(3x^2 - 20x^4) = \frac{3x^2 - 20x^4}{x^3 - 4x^5}$$

$$= \frac{\cancel{x^2}(3 - 20x^2)}{\cancel{x^2}(x - 4x^3)}$$

$$f(x) = x^3\sec(\ln x^2)$$

$$f'(x) = x^{\cancel{3}\,1}\cdot\sec(\ln x^2)\tan(\ln x^2)\cdot\frac{1}{\cancel{x^2}}\cdot 2x + \sec(\ln x^2)\cdot 3x^2$$

$f(x) = \log_8 3x^7$

$f(x) = \frac{\ln 3x^7}{\ln 8} = \frac{1}{\ln 8} \cdot \ln 3x^7$

$f'(x) = \frac{1}{\ln 8} \cdot \frac{1}{\cancel{3x^7}\,x} \cdot \overset{7}{\cancel{21}}\cancel{x^6}$

$= \frac{7}{\ln 8} \cdot \frac{1}{x}$

Change of Base Formula

$\log_b a = \frac{\ln a}{\ln b}$

$$f(x) = x^{x^2} = e^{\ln x^{x^2}}$$

$$= e^{x^2 \cdot \ln x}$$

$$e^{\ln x} = x$$

$$f'(x) = \underbrace{e^{x^2 \cdot \ln x}} \cdot \left[x^2 \cdot \frac{1}{\cancel{x}} + \ln x \cdot 2x\right]$$

$$= x^{x^2} \cdot \left[x + 2x \ln x\right]$$

$$= x^1 \cdot x^{x^2}\left[1 + 2\ln x\right]$$

$$= \boxed{x^{x^2+1}\left[1 + 2\ln x\right]}$$

$$f(x) = x^{\cos x} = e^{\ln x^{\cos x}} = e^{\cos x \cdot \ln x}$$

$$f'(x) = e^{\cos x \cdot \ln x} \cdot \left[\cos x \cdot \frac{1}{x} + \ln x \cdot -\sin x\right]$$

$$= x^{\cos x}\left[\frac{\cos x}{x} - \frac{x \cdot \ln x \sin x}{x \cdot 1}\right]$$

$$= x^{\cos x}\left[\frac{\cos x - x \ln x \sin x}{x}\right]$$

$$= \frac{x^{\cos x}}{x^1}\left[\cos x - x \ln x \sin x\right]$$

$$= x^{\cos x - 1}\left[\cos x - x \ln x \sin x\right]$$