Inverse Trig Functions

Find $\frac{dy}{dx}$.

$$y = \sin^{-1} x$$

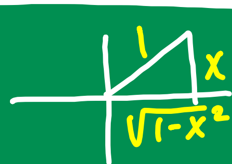
$$\frac{y}{r} = \frac{x}{1} = \sin y$$

$$1 = \cos y \frac{dy}{dx}$$

$$\frac{1}{\cos y} = \frac{dy}{dx}$$

$$\frac{1}{\frac{\sqrt{1-x^2}}{1}} = \frac{dy}{dx}$$

$$\frac{1}{\sqrt{1-x^2}} = \frac{dy}{dx}$$



$\frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$f(x) = \sin^{-1}(7x^5)$$

$$f'(x) = \frac{1}{\sqrt{1-(7x^5)^2}} \cdot 35x^4 = \frac{35x^4}{\sqrt{1-49x^{10}}}$$

$$f(x) = \boxed{\csc^{-1}(x^4)} \cdot \boxed{\tan^{-1}(\ln x^2)}$$

$$f'(x) = \left[\csc^{-1}(x^4)\right] \cdot \left[\frac{1}{1+(\ln x^2)^2} \cdot \frac{1}{\cancel{x^2}} \cdot 2\cancel{x}\right] + \left[\tan^{-1}(\ln x^2)\right] \cdot \frac{-1}{\cancel{x^4}\sqrt{x^8-1}} \cdot 4\cancel{x^3}$$

$$= \csc^{-1}(x^4)\left(\frac{2}{x}\right)\left(\frac{1}{1+(\ln x^2)^2}\right) - \frac{4\tan^{-1}(\ln x^2)}{\sqrt{x^8-1}}$$

L'HOPITAL'S RULE

– Indeterminate Forms

$$\frac{0}{0},\ \frac{\infty}{\infty},\ 0\cdot\infty$$

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \frac{0}{0}$$

$$\lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = 4$$

$$\lim_{x\to 2} \frac{2x}{1} = 4$$

$$\lim_{x\to \#} \frac{f(x)}{g(x)} = \lim_{x\to \#} \frac{f'(x)}{g'(x)}$$

must be $\frac{0}{0}$ OR $\frac{\infty}{\infty}$

$$\lim_{x\to 1} \frac{x^3-3x^2+5x-3}{x^2+x-2} = \frac{0}{0}$$

$$\lim_{x\to 1} \frac{3x^2-6x+5}{2x+1} = \frac{3-6+5}{2+1} = \frac{2}{3}$$

$$\lim_{x \to 0} \frac{e^x - 1 - x}{\cos(2x) - 1} = \frac{1-1-0}{1-1} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{e^x - 1}{\begin{array}{c} -\sin(2x)\cdot 2 \\ -2\sin(2x) \end{array}} = \frac{1-1}{0 \cdot 2} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{e^x}{-2\cos(2x)\cdot 2} = \frac{1}{-2 \cdot 1 \cdot 2} = \frac{1}{-4}$$

$$\lim_{x\to 0^+} \frac{1-\ln x}{e^{1/x}} = \frac{1+{}^{+}\infty}{e^{+\infty}} = \frac{\infty}{\infty} \qquad x^{-1}$$

$$\lim_{x\to 0^+} \frac{-\frac{1}{x}}{e^{1/x}\cdot\frac{-1}{x^2}}$$

$$\lim_{x\to 0^+} +\frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{2}\,1}}{+e^{1/x}}$$

$$\lim_{x\to 0^+} \frac{x}{e^{1/x}} = \frac{0}{\infty} = 0$$

$$\lim_{x\to -\infty} e^x = 0$$

$$\lim_{x\to +\infty} e^x = +\infty$$

$$\lim_{x\to 0^+} \ln x = -\infty$$

$$\lim_{x\to +\infty} \ln(x) = +\infty$$