SPECIAL DERIVATIVES

- Implicit Differentiation $x^2y + 3y^4 = 4xy^8$

Explicit $y = x^3 + x^2 + 7x - 4$

* variables mixed

* When y has a power(s)

Pretend $y = 3x^2 + 7x - 4$

$$\frac{dy}{dx} = 6x + 7$$

Find $\frac{dy}{dx}$.

$$y^2 + x^3 + y^3 = 5$$

$$(3x^2+7x-4)^2 + x^3 + (3x^2+7x-4)^3 = 5$$

$$2(3x^2+7x-4)^1 \cdot (6x+7) + 3x^2 + 3(3x^2+7x-4)^2 \cdot (6x+7) = 0$$

$$2y \cdot \frac{dy}{dx} + 3x^2 + 3y^2 \cdot \frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -3x^2$$

$$\frac{dy}{dx}\left[2y + 3y^2\right] = -3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2}{2y + 3y^2}}$$

Find $\frac{dy}{dx}$ ← Find deriv. of "Normal"

$(3x^2y^3) + 4y^5 = 6\sin y + 8x^5$

$$\left[3x^2 \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot 6x\right] + 20y^4 \frac{dy}{dx} = 6\cos y \frac{dy}{dx} + 40x^4$$

$$= 9x^2y^2\frac{dy}{dx} + 6xy^3 \quad + 20y^4\frac{dy}{dx} = 6\cos y\frac{dy}{dx} + 40x^4$$

$$\frac{dy}{dx}\left[9x^2y^2 + 20y^4 - 6\cos y\right] = 40x^4 - 6xy^3$$

$$\boxed{\frac{dy}{dx} = \frac{40x^4 - 6xy^3}{9x^2y^2 + 20y^4 - 6\cos y}}$$

Find the equation of the tangent line at (1,0)

$$m = \frac{40\cdot 1 - \cancel{6(1)(0)^3}}{\cancel{9(1)^2(0)^2} + \cancel{20(0)^4} - 6\cos 0} = \frac{40}{-6} = -\frac{20}{3}$$

$$y - 0 = -\frac{20}{3}(x-1)$$

$$\boxed{y = -\frac{20}{3}x + \frac{20}{3}}$$

Find $\frac{da}{dp}$. ← normal

$$3r^7 + 6a^5 - 4p = p^7$$

$$21r^6 \cdot \frac{dr}{dp} + 30a^4 \frac{da}{dp} - 4 = 7p^6$$

$$\frac{\cancel{30a^4}\frac{da}{dp}}{\cancel{30a^4}} = \frac{7p^6 - 21r^6\frac{dr}{dp} + 4}{30a^4}$$

Find $\frac{dy}{dt}$. ← normal

$$4x^3 + 2y^5 = \frac{x}{y}$$

$$y^2\left[12x^2\frac{dx}{dt} + 10y^4\frac{dy}{dt} = \frac{y\cdot\frac{dx}{dt} - x\cdot\frac{dy}{dt}}{y^2}\right]$$

$$12x^2y^2\frac{dx}{dt} + 10y^6\frac{dy}{dt} = y\frac{dx}{dt} - x\frac{dy}{dt}$$

$$\frac{dy}{dt}\left[10y^6 + x\right] = y\frac{dx}{dt} - 12x^2y^2\frac{dx}{dt}$$

$\frac{dx}{dt}$

$$\frac{dy}{dt} = \frac{y\frac{dx}{dt} - 12x^2y^2\frac{dx}{dt}}{10y^6 + x}$$