More L'Hopital's Rule

$$\lim_{x\to 0^+} x^2 \cdot \ln x = 0$$

$0 \cdot -\infty$

$$\lim_{x\to 0^+} \frac{\ln x}{x^{-2}} = \frac{-\infty}{\infty}$$

$\frac{1}{x^2}$

$$\lim_{x\to 0^+} \frac{\frac{1}{\cancel{x}}}{\frac{-2}{x^3}} \qquad \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{3}\,2}}{-2}$$

$$\lim_{x\to 0^+} \frac{x^2}{-2} = \frac{0}{-2} = \boxed{0}$$

Indeterminate forms:

$\frac{0}{0}$, $\frac{\infty}{\infty}$

$0 \cdot \infty$, $\infty - \infty$, 0^0, 1^∞, ∞^0

Must be rearranged to be $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$\lim_{x\to 0^+}\left(\csc x - \frac{1}{x}\right) = \infty - \infty$$

$$\lim_{x\to 0^+}\left(\frac{x\cdot 1}{x\sin x} - \frac{1\,\sin x}{x\sin x}\right)$$

$$\lim_{x\to 0^+}\frac{x-\sin x}{x\cdot\sin x} = \frac{0-0}{0\cdot 0}$$

$$\lim_{x\to 0^+}\frac{1-\cos x}{x\cdot\cos x+\sin x\cdot 1} = \frac{1-1}{0+0} = \frac{0}{0}$$

$$\lim_{x\to 0^+}\frac{+\ +\sin x}{x\cdot -\sin x+\cos x\cdot 1+\cos x}$$

$$\lim_{x\to 0^+}\frac{\sin x}{-x\sin x+2\cos x} = \frac{0}{0+2\cdot 1} = \frac{0}{2} = 0$$

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$$\lim_{x\to\infty} x^{\frac{1}{x}} = \infty^{\frac{1}{\infty}} = \infty^0$$

$$\lim_{x\to\infty} e^{\ln x^{1/x}}$$

$$\lim_{x\to\infty} e^{\frac{\ln x}{x}}$$

$$e^{\lim_{x\to\infty}\frac{\ln x}{x}}$$

$$\lim_{x\to\infty}\frac{\ln x}{x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{\frac{1}{x}}{1} = \frac{1}{\infty} = 0$$

$$e^0 = \boxed{1}$$

$$\lim_{x\to 0^+}(\csc x)^{\sin x} = \infty^0$$

$$\lim_{x\to 0^+} e^{\ln(\csc x)^{\sin x}}$$

$$\lim_{x\to 0^+} e^{\sin x\cdot\ln(\csc x)}$$

$$\lim_{x\to 0^+}\frac{\ln(\csc x)}{\csc x} = \frac{\ln(\infty)}{\infty} = \frac{\infty}{\infty}$$

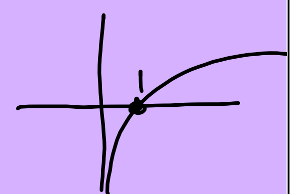
$$\lim_{x\to 0^+}\frac{\frac{1}{\csc x}\cdot\cancel{-\csc x\cot x}}{\cancel{-\csc x\cot x}} = \frac{1}{\infty} = 0$$

$$e^0 = \boxed{1}$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = \left(1+\overset{0}{\cancel{\frac{1}{\infty}}}\right)^{\infty} = 1^{\infty}$$

$$\lim_{x\to\infty} e^{x\cdot\ln\left(1+\frac{1}{x}\right)} \qquad x^{-1}$$

$$e^{\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}} = \frac{\ln\left(1+\frac{1}{\infty}\right)}{\frac{1}{\infty}} = \frac{\ln 1}{0} = \frac{0}{0}$$



$$\lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x}}\cdot\cancel{-\frac{1}{x^2}}}{\cancel{-\frac{1}{x^2}}} = \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1} = 1$$

$$\boxed{e^{1}}$$

Steps for derivative of $x^{f(x)}$.

1) Rewrite as $e^{\ln x^{f(x)}} = e^{f(x)\cdot \ln x}$
2) Perform derivative on new function (product rule!)
3) Change $e^{f(x)\cdot \ln x}$ back to original expression and simplify (pull out common factors)

$\ln(x^2-2x)$