# LOGARITHMS – inverses of exponential functions

$y = b^x$

$b > 0,\ b \neq 1$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

$y = \log_b x$

$b > 0,\ b \neq 1$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

purpose:
to solve for exponents!

$y = b^x$

$\log_b x = \cancel{\log_b} b^y$

$\log_b x = y$

John Napier
find distances to planet

$y = \log(x-3)$

$-$ $+$

0 3

$(3, \infty)$

$y = \log_{81}(25 - x^2)$

$-$ $+$ $-$

-5 0 5

$(-5, 5)$

Natural Logs

$\log_e x = \ln x$

Common Logs

$\log_{10} x = \log x$

$y = 2^x$

| $x$ | $y$ |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |

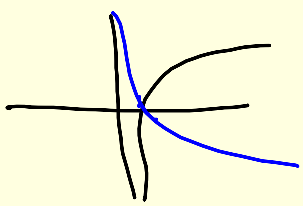
$y = \log_2 x$

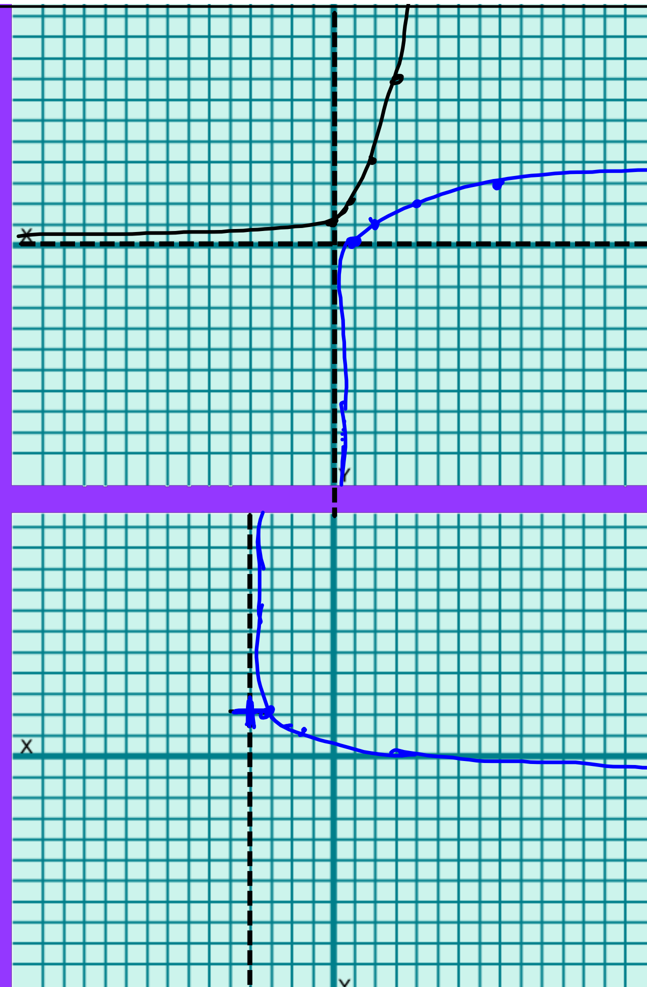
| $x$ | $y$ |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 4 | 2 |
| 8 | 3 |

$y = -\ln(x+4) + 2$

left 4  up 2

| | |
|---|---|
| $e^0 = 1$ | 0 |
| $e^1 = 2.7$ | $-1$ |
| $e^2 = 7.4$ | $-2$ |





$\log_9 81 = \log_9 9^2 = 2$

$\log_6 \frac{1}{36} = \log_6 6^{-2} = -2$

$\log_7 \sqrt[5]{49} = \log_7 \sqrt[5]{7^2} = \log_7 7^{2/5} = \frac{2}{5}$

$\log_{10} 1000 = \log_{10} 10^3 = 3$

$\ln_e e^{3178} = 3178$

$\ln \frac{1}{\sqrt[2]{e^3}} = \ln e^{-3/2} = -3/2$

$e^{\ln_e 56} = 56$

$e^{2 \ln 8} = e^{\ln 8^2} = 64$

# Solving Log Equations

## Properties of Logs

$$\log_b m + \log_b n = \log_b(m \cdot n)$$

$$\log_b m - \log_b n = \log_b\left(\frac{m}{n}\right)$$

$$\log_b m^p = p \cdot \log_b m$$

$$\log_4 8 = x$$

$$4^{\log_4 8} = 4^x$$

Exponentiate!

$$8 = 4^x$$

$$2^3 = 2^{2x}$$

$$3 = 2x$$

$$3/2 = x$$

$$\log_x 64 = 3$$

$$x^{\log_x 64} = x^3$$

$$\sqrt[3]{64} = \sqrt[3]{x^3}$$

$$\boxed{4 = x}$$

$$\log_7(x-2) + \log_7(2x-3) = 2\log_7 x$$

$$(x-2)(2x-3)$$

$$\log_7(2x^2-7x+6) = \log_7 x^2$$

$$7^{\log_7(2x^2-7x+6)} = 7^{\log_7 x^2}$$

$$2x^2-7x+6 = x^2$$

$$-x^2$$

$$x^2-7x+6=0$$

$$(x-1)(x-6)=0$$

$$x = \cancel{1},\ 6$$

$$\log x - \log 2 = 3$$

$$10^{\log\left(\frac{x}{2}\right)} = 10^3$$

$$\frac{x}{2} = 1000$$

$$\boxed{x = 2000}$$