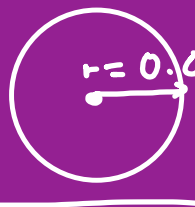
Related Rates

$\frac{mi}{h}$ $\frac{m}{s}$ $\frac{cm}{h}$ $\frac{gal}{min}$ $\frac{rad}{sec}$

- rate of one part of the situation impacts the rate of another part.

Example 1



$r = 4$ in

$\frac{d}{dt}\left[A = \pi r^2\right]$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(4\ in)(0.02\ \frac{in}{sec})$

$\frac{dA}{dt} = 0.16\pi\ \frac{in^2}{sec}$

$\approx \boxed{0.5\ \frac{in^2}{sec}}$

1) Draw a picture
2) Label with Variables (changing) & Constants (not changing)
3) Set up a formula
4) Do derivative with respect to time using implicit differentiation.
5) Identify the rate to be found.
6) Fill in values & solve.

2) 

$\frac{dV}{dt} = 0.2 \frac{m^3}{min}$

Find $\frac{dr}{dt}$

$S.A = 0.64\pi \, m^2$

$\frac{4\pi r^2}{4\pi} = \frac{0.64\pi}{4\pi}$

$\sqrt{r^2} = \sqrt{0.16}$

$r = 0.4$

$\frac{d}{dt}\left[V = \frac{4}{3}\pi r^3\right]$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-0.2 = 4\pi(0.4)^2 \frac{dr}{dt}$

$\frac{-0.2}{0.64\pi} = \frac{0.64\pi}{0.64\pi} \frac{dr}{dt}$

$\frac{m^3}{min} \cdot \frac{1}{m^2}$

$-0.0994 = \frac{dr}{dt}$

$\approx -0.1 \frac{m}{min}$

3/

$h = 2r \Rightarrow \frac{h}{2} = r$

Find rate of height when 10 ft high

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 \cdot h$$

$$V = \frac{1}{3}\pi \cdot \frac{h^2}{4} \cdot h$$

$$\frac{d}{dt}\left[V = \frac{1}{12}\pi h^3\right]$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$5 = \frac{1}{4}\pi (10)^2 \frac{dh}{dt}$$

$$\frac{5}{25\pi} = \frac{25\pi}{25\pi}\frac{dh}{dt}$$

$$\boxed{\frac{1}{5\pi}\ \frac{ft}{min} = \frac{dh}{dt}}$$

$5^2 + 12^2 = y^2$

$25 + 144 = y^2$

$169 = y$

$13 = y$

$\frac{d}{dt}\left[25 + x^2 = y^2\right]$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$2(12)\frac{dx}{dt} = 2(13)(-4)$

$24\frac{dx}{dt} = -104$

$\frac{24}{24}\frac{dx}{dt} = \frac{-104}{24}$

$\frac{dx}{dt} = -\frac{13}{3}$ ft/min