Special Derivatives Review

List:

$\frac{d}{dx}\sin^{-1}x$ $\quad \frac{d}{dx}e^x = e^x$ $\quad \log_b a = \frac{\ln a}{\ln b}$

$\frac{d}{dy}\tan^{-1}x$ $\quad \frac{d}{dx}\ln x = \frac{1}{x}$ $\quad \lim_{x\to\infty} e^x = \infty$ $\quad \lim_{x\to\infty} \ln x = \infty$

$\frac{d}{dx}\sec^{-1}x$ $\quad \frac{d}{dx}a^x = \ln a \cdot a^x$ $\quad \lim_{x\to-\infty} e^x = 0$ $\quad \lim_{x\to 0^+} \ln x = -\infty$

Implicit Differentiation

Find $\frac{dy}{dx}$.

$x^2y^3 + 3x^2 = 4 - 8y^5$

product

$x^2 \cdot 3y^2\frac{dy}{dx} + y^3 \cdot 2x + 6x = -40y^4\frac{dy}{dx}$

$3x^2y^2\frac{dy}{dx} + 2xy^3 + 6x = -40y^4\frac{dy}{dx}$

$\frac{dy}{dx}\left(3x^2y^2 + 40y^4\right) = -2xy^3 - 6x$

$\frac{dy}{dx} = \frac{-2x^3y - 6x}{3x^2y^2 + 40y^4}$

Find eq. of tangent line at $(\underset{x}{1}, \underset{y}{-1})$

$m = \frac{-2(1)(-1) - 6(1)}{3(1)(1) + 40(1)} = \frac{-4}{43}$

$4m^3 + 7p^5 = 8n^5 - 7$ $\quad$ Find $\frac{dm}{dp}$

$12m^2\frac{dm}{dp} + 35p^4 = 40n^4\frac{dn}{dp}$

$\frac{12m^2\frac{dm}{dp}}{12m^2} = \frac{40n^4\frac{dn}{dp} - 35p^4}{12m^2}$

DERIVATIVES

$$y = x^4 \cdot 7^{\sin x}$$

$$y' = x^4 \cdot \ln 7 \cdot 7^{\sin x} \cdot \cos x + 7^{\sin x} \cdot 4x^3$$

$$= x^3 7^{\sin x} \left[x \cdot \ln 7 \cdot \cos x + 4 \right]$$

$$y = \sec^{-1}(\sec x) = x$$

$$y' = \frac{1}{|\sec x| \sqrt{\sec^2 x - 1}} \cdot \sec x \tan x = \frac{\sec x \tan x}{\sec x \sqrt{\sec^2 x - 1}}$$

$$= \frac{\tan x}{\sqrt{\tan^2 x}} = \frac{\tan x}{\tan x} = 1$$

$$f(x) = x^{e^{x^2}} = e^{\ln x^{e^{x^2}}} = e^{e^{x^2}\cdot \ln x}$$

$$f'(x) = e^{e^{x^2}\cdot \ln x} \cdot \left[e^{x^2}\cdot \frac{1}{x} + \ln x \cdot e^{x^2}\cdot 2x \right]$$

$$= x^{e^{x^2}} \cdot e^{x^2} \left[\frac{1}{x} + x \cdot \frac{2x \ln x}{x \cdot 1} \right]$$

$$= x^{e^{x^2}} \cdot e^{x^2} \left[\frac{1 + 2x^2 \ln x}{x} \right]$$

$$f(x) = \frac{\log x^3}{6^{x^2}} = \frac{\frac{\ln x^3}{\ln 10}}{6^{x^2}} = \frac{\frac{1}{\ln 10}\ln x^3}{6^{x^2}}$$

$$f'(x) = \frac{6^{x^2}\cdot\frac{1}{\ln 10}\cdot\frac{1}{x^3}\cdot 3x^2 - \frac{1}{\ln 10}\ln x^3 \cdot \ln 6 \cdot 6^{x^2}\cdot 2x}{(6^{x^2})^2}$$

$$= \frac{1}{\ln 10}\,6^{x^2}\left[\frac{3}{x} - \frac{2\ln 6 \cdot x^2 \ln x^3}{x}\right]$$

$$= \frac{6^{x^2}}{\ln 10}\left[\frac{3 - 2\ln 6 \cdot x^2 \cdot \ln x^3}{x}\right] \cdot \frac{1}{6^{2x^2}} =$$

$$\frac{1}{\ln 10 \cdot 6^{x^2}}\left[\frac{3 - 2\ln 6 \cdot x^2 \cdot \ln x^3}{x}\right]$$