CONIC SECTIONS HANDOUT
PARABOLAS

Find the midpoint and distance between the given points.

1. $(-2,8),\ (6,0)$

2. $(-7,2),\ \left(-\frac{11}{2},4\right)$

Find all parts of each parabola and graph.

3. $x=\frac{1}{6}(y+2)^2$

4. $x=-\frac{1}{4}(y-8)^2+4$

5. $y=\frac{1}{2}(x-3)^2+5$

6. The headlights on a car contain parabolic reflectors. A special light bulb with two filaments is used to produce the high and low beams. The filament placed at the focus produces the high beam. Given the equation of the parabolic reflector is $y=\frac{1}{10}x^2$ inches, where should the filament be placed?

7. The cross section of a television antenna dish is a parabola with equation $y=\frac{1}{16}x^2$ feet. If the receiver is located at the focus, how far is the receiver from the base of the dish?

CIRCLES

Determine the center and radius of each circle.

1. $x^2+y^2=169$

2. $(x-1)^2+(y+5)^2=25$

3. $x^2-6x+y^2=0$

4. $2x^2+2y^2+16x+4y=8$

5. $3x^2+3y^2+18x-6y=42$

6. $x^2+y^2-8y=9$

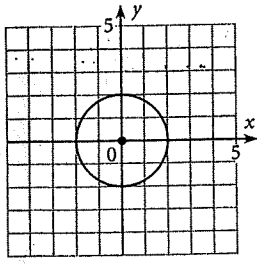
Graph each circle.

7. $x^2+y^2=25$

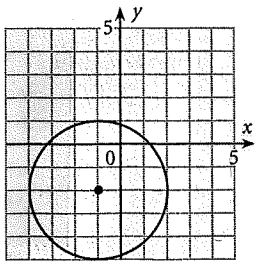
8. $(x-2)^2+(y+5)^2=9$

Write the equation of each circle in standard form.

9.



10.



11. The endpoints of the diameter are at (-2,-3) and at (4,5)

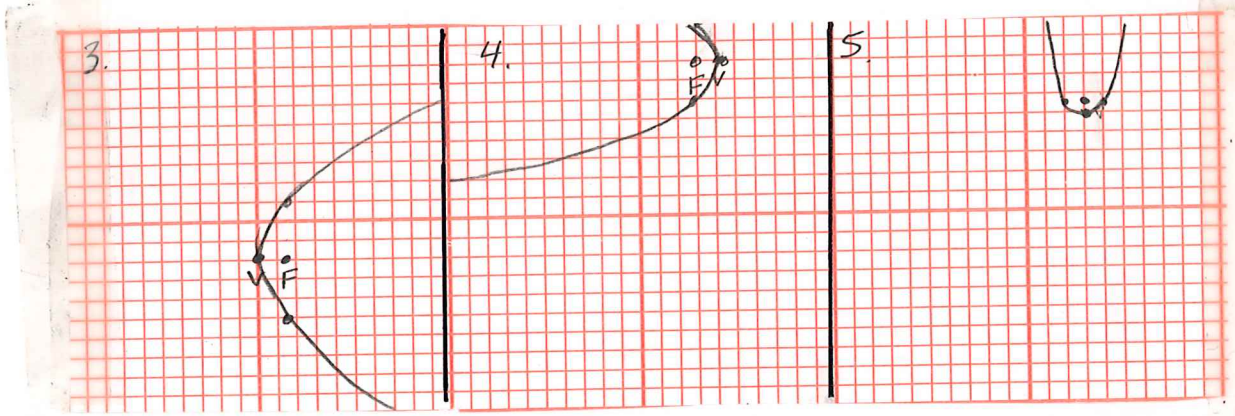
12. The circle has center (-4,9) and is tangent to the y-axis.

13. Earth's orbit around the sun is nearly circular. The radius of the orbit is approximately 9.3×10^7 miles. If the center of the sun is located at (0,0), write an equation for the orbit of Earth around the sun.

PARABOLA ANSWERS

1. $(2,4)$; $8\sqrt{2}$
2. $\left(-\frac{25}{4},3\right)$; $\frac{5}{2}$

	3.	4.	5.
Vertex	$(0,-2)$	$(4,8)$	$(3,5)$
Axis of Symmetry	$y=-2$	$y=8$	$x=3$
Direction	Right	Left	Up
Focus	$\left(\frac{3}{2},-2\right)$	$(3,8)$	$(3,5\frac{1}{2})$
Latus Rectum	6	4	2

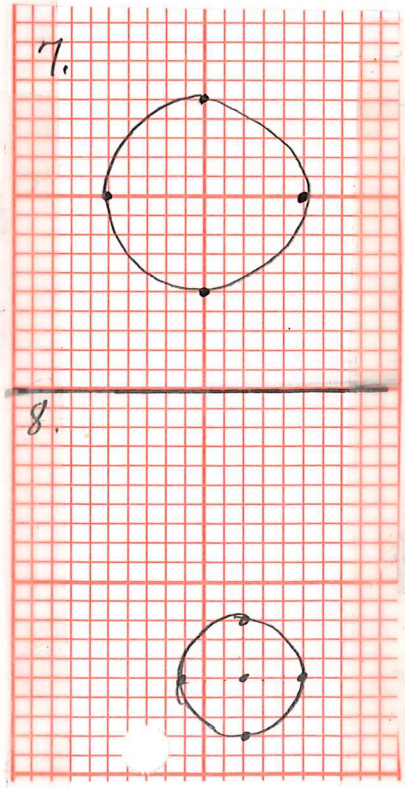


6. 2.5 in.
7. 4 ft.

CIRCLE ANSWERS

1. $(0,0)$; $r=13$
2. $(1,-5)$; $r=5$
3. $(3,0)$; $r=3$
4. $(-4,-1)$; $r=\sqrt{21}$
5. $(-3,1)$; $r=2\sqrt{6}$
6. $(0,4)$; $r=5$

7-8. See graphs at right



9. $x^2+y^2=4$
10. $(x+1)^2+(y+2)^2=9$
11. $(x-1)^2+(y-1)^2=25$
12. $(x+4)^2+(y-9)^2=16$
13. $x^2+y^2=8.649\times10^{15}$

CONIC SECTIONS HANDOUT
ELLIPSES

Find the center, a, b, c, orientation, length of major & minor axes, foci, and eccentricity of the graph of each ellipse. Draw the graph.

1. $\frac{(x-2)^2}{64}+\frac{(y+1)^2}{16}=1$

2. $\frac{(x+4)^2}{9}+\frac{(y-3)^2}{36}=1$

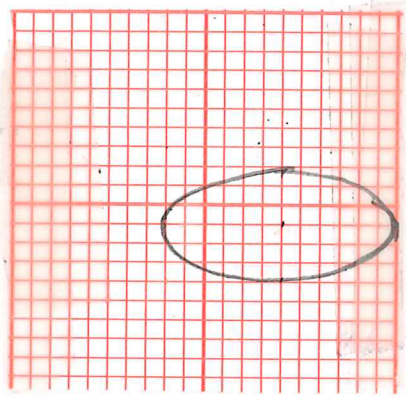
3. $9x^2+4y^2-18x+24y+9=0$

4. $4x^2+25y^2+40x=0$

5. $49x^2+4y^2+392x+16y+604=0$

Write the equation for each ellipse in standard form.

6.



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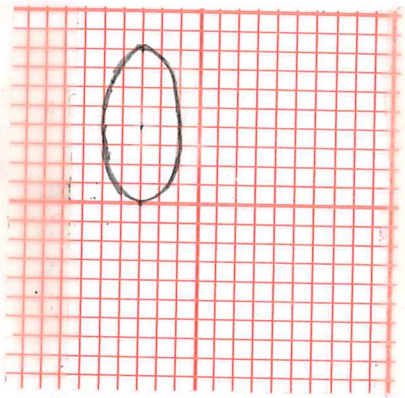


8. Earth's orbit around the sun is actually elliptical, with the sun at one focus. The length of the major axis is 186 million miles, and the two foci are 3.2 million miles apart.
   a. What is the length of the minor axis of the ellipse?
   b. Sketch a graph of Earth's orbit, showing the location of the sun.
   c. What is the greatest distance between Earth and the sun?
   d. What is the smallest distance between Earth and the sun?

9. An arch is to be built in the shape of a semi-ellipse. The opening is to have a height of 50 ft at the center and a span of 200 ft. Write an equation of an ellipse that represents this arch.

10. In "whispering galleries" a sound made at one focus can be clearly heard at the other focus, even though very little can be heard by someone in between. Suppose an elliptical room measures 320 ft long and 150 ft wide. How far would the listener have to be from the source of the sound in order to hear it?

ELLIPSE ANSWERS

1. $(2,-1)$; $a=8, b=4,\ c=4\sqrt{3}$; horizontal; major $=16,$ minor $=8$; $\left(2\pm4\sqrt{3},-1\right)$; $\frac{\sqrt{3}}{2}\approx0.87$

2. $(-4,3)$; $a=6, b=3,\ c=3\sqrt{3}$; vertical; major $=12,$ minor $=6$; $\left(-4,3\pm3\sqrt{3}\right)$; $\frac{\sqrt{3}}{2}\approx0.87$

3. $(1,-3)$; $a=3, b=2,\ c=\sqrt{5}$; vertical; major $=6,$ minor $=4$; $\left(1,-3\pm\sqrt{5}\right)$; $\frac{\sqrt{5}}{3}\approx0.75$

4. $(-5,0)$; $a=5, b=2,\ c=\sqrt{21}$; horizontal; major $=10,$ minor $=4$; $\left(-5\pm\sqrt{21},0\right)$; $\frac{\sqrt{21}}{5}\approx0.92$

5. $(-4,-2)$; $a=7, b=2,\ c=3\sqrt{5}$; vertical; major $=14,$ minor $=4$; $\left(-4,-2\pm3\sqrt{5}\right)$; $\frac{3\sqrt{5}}{7}\approx0.96$

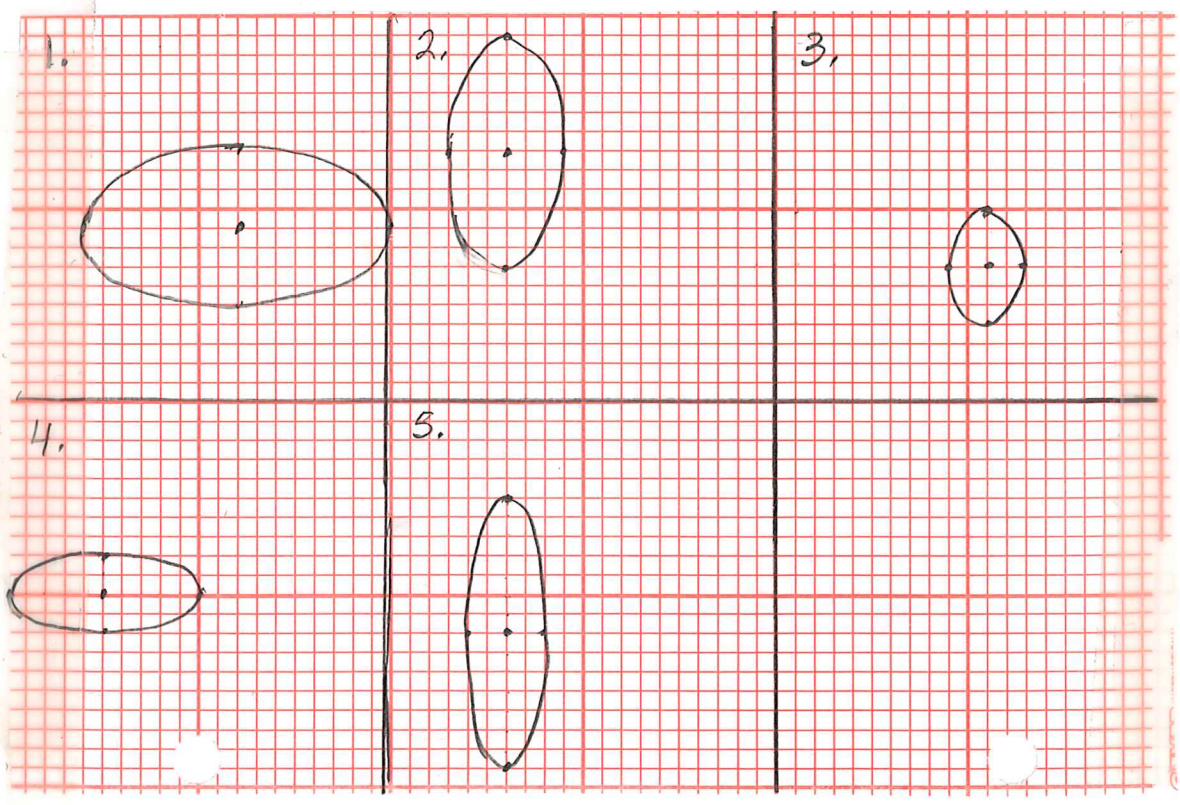
6. $\frac{(x-4)^2}{36}+\frac{(y+1)^2}{9}=1$

7. $\frac{(x+3)^2}{4}+\frac{(y-4)^2}{16}=1$

8. a) 185.972 million miles
   c) 94.6 million miles
   d) 91.4 million miles

9. $\frac{x^2}{10,000}+\frac{y^2}{2500}=1$

10. $10\sqrt{799}\approx282.7$ ft



CONIC SECTIONS HANDOUT
HYPERBOLAS

Find the center, a, b, c, orientation, vertices, foci, and slopes of the asymptotes of each hyperbola. Draw the graph.

1. $\frac{(x+1)^2}{4} - y^2 = 1$

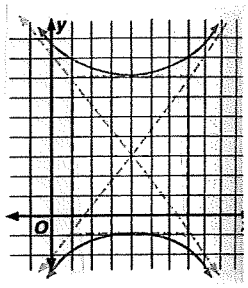
2. $\frac{(y+2)^2}{25} - \frac{(x-5)^2}{64} = 1$

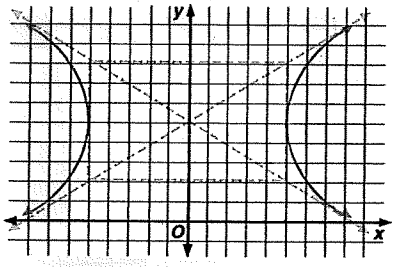
3. $25x^2 - 4y^2 - 50x - 16y - 91 = 0$

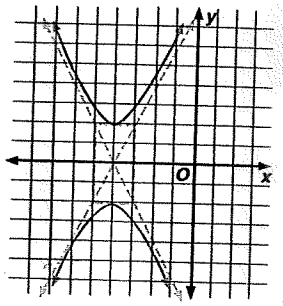
4. $y^2 - 4x^2 - 2y - 16x - 31 = 0$

5. $25y^2 - 9x^2 - 100y - 72x - 269 = 0$

Write the equation for each hyperbola in standard form.

6. 

7. 

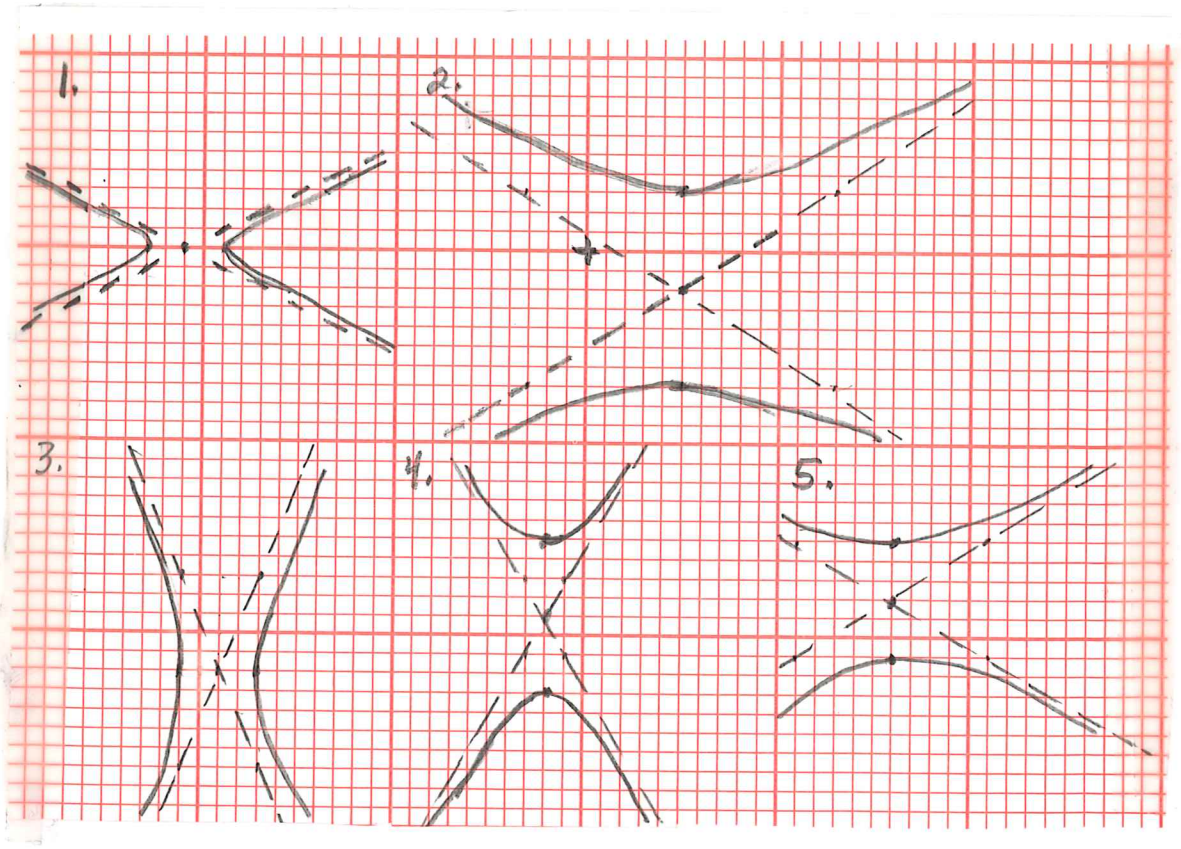
8. 

9. Suppose that part of a comet's path approximates a branch of a hyperbola with the sun at the focus. At its closest approach to the sun, the comet is 25 million miles away. Assume that the center of its path is at (0,0) and its vertex is 9 million miles from the center. Write an equation for a hyperbola that models the path of this comet.

10. A hyperbolic mirror is shaped like one branch of a hyperbola. It reflects any light directed toward one focus of the hyperbola through the other focus. Suppose that the center of a hyperbolic mirror is at (0,0), the vertex of the mirror's branch is at (3,0), and the slopes of the asymptotes are $\pm\frac{4}{3}$. Write an equation that models the surface of the mirror.

HYPERBOLA ANSWERS

1. $(-1,0)$; $a=2, b=1$, $c=\sqrt{5}$; horizontal; $(-3,0),(1,0)$; $\left(-1\pm\sqrt{5},0\right)$; $\pm\frac{1}{2}$
2. $(5,-2)$; $a=5, b=8$, $c=\sqrt{89}$; vertical; $(5,-7),(5,3)$; $\left(5,-2\pm\sqrt{89}\right)$; $\pm\frac{5}{8}$
3. $(1,-2)$; $a=2, b=5$, $c=\sqrt{29}$; horizontal; $(-1,-2)$, $(3,-2)$; $\left(1\pm\sqrt{29},-2\right)$; $\pm\frac{5}{2}$
4. $(-2,1)$; $a=4, b=2$, $c=2\sqrt{5}$; vertical; $(-2,-3),(-2,5)$; $\left(-2,1\pm2\sqrt{5}\right)$; ±2
5. $(-4,2)$; $a=3, b=5$, $c=\sqrt{34}$; vertical; $(-4,-1),(-4,5)$; $\left(-4,2\pm\sqrt{34}\right)$; $\pm\frac{3}{5}$
6. $\frac{(y-3)^2}{16}-\frac{(x-4)^2}{9}=1$
7. $\frac{x^2}{25}-\frac{(y-5)^2}{9}=1$
8. $\frac{y^2}{4}-\frac{(x+4)^2}{1}=1$
9. $\frac{x^2}{81}-\frac{y^2}{1075}=1$
10. $\frac{x^2}{9}-\frac{y^2}{16}=1$



CONIC SECTIONS HANDOUT
IDENTIFYING CONIC SECTIONS

Determine whether the graph of each equation is a parabola, a circle, an ellipse, or a hyperbola.

1. $\frac{(x-4)^2}{9}-\frac{(y+2)^2}{9}=1$
2. $\frac{(y-7)^2}{3}+\frac{(x+2)^2}{2}=1$
3. $x=(y+4)^2-6$
4. $\frac{x^2}{25}+\frac{(y+8)^2}{25}=1$
5. $x^2=121-y^2$
6. $16x^2-9y^2-48x-54y-8=0$
7. $x^2=8y$
8. $25y^2+9x^2-50y-54x=119$
9. $x+2=x^2+y$
10. $3y^2+24y+41=2x-x^2$
11. $6x^2-36x+6y^2-18y=99$
12. $25y+5y^2-125=9x^2-72x$

QUADRATIC SYSTEMS OF EQUATIONS

Solve each system of equations by hand.

1. $y^2-x^2=9$
   $2x-y=3$
2. $x^2+y^2=74$
   $y^2-x^2=24$
3. $2x^2+y=0$
   $x^2+y^2=5$
4. $5x^2+y^2=30$
   $y^2-16=9x^2$

Use a graphing utility to solve each system of equations. Round answers to the nearest hundredth.

5. $x-y=1$
   $x^2=2y+3$
6. $3x^2-y^2=9$
   $x^2+2y^2=10$

IDENTIFY CONICS ANSWERS

1. Hyperbola
2. Ellipse
3. Parabola
4. Circle
5. Circle
6. Hyperbola
7. Parabola
8. Ellipse
9. Parabola
10. Ellipse
11. Circle
12. Hyperbola

QUADRATIC SYSTEMS ANSWERS

1. $(0,-3),\ (4,5)$
2. $(5,7),\ (5,-7),\ (-5,7),\ (-5,-7)$
3. $(1,-2),\ (-1,-2)$
4. $(1,5),\ (1,-5),\ (-1,5),\ (-1,-5)$
5. $(2.41,1.41),\ (-0.41,-1.41)$
6. $(2,1.73),\ (2,-1.73),\ (-2,1.73),\ (-2,-1.73)$