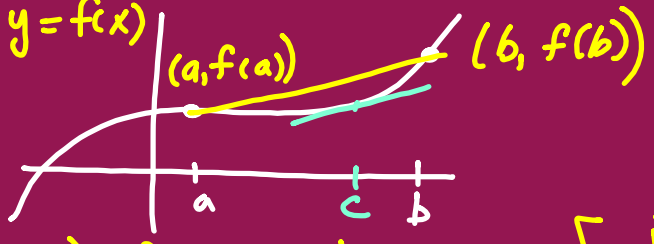
November 10, 2021

Curve Sketching

Mean Value Theorem



1) f is continuous on $[a,b]$
2) f is differentiable on (a,b)
3) $f'(c) = \frac{f(b)-f(a)}{b-a}$

$f(x) = x^3 - 3x^2 + 2x \quad [0, 2]$

$f'(x) = 3x^2 - 6x + 2$

$3c^2 - 6c + 2 = \frac{f(2)-f(0)}{2-0}$

$3c^2 - 6c + 2 = \frac{0-0}{2-0}$

$3c^2 - 6c + 2 = 0$

$c = \frac{6 \pm \sqrt{36 - 4(3)(2)}}{2(3)}$

$= \frac{6 \pm \sqrt{12}}{6}$

$c = \frac{6 \pm 2\sqrt{3}}{6} = \frac{3 \pm \sqrt{3}}{3}$

$c \approx 1.6$ or 0.4 ← both work

$c = \frac{3+\sqrt{3}}{3}$ or $\frac{3-\sqrt{3}}{3}$

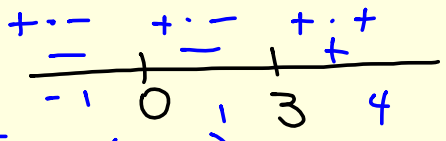
$f(x) = x^4 - 4x^3 + 10$

CRIT PTS

$f'(x) = 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$x = 0 \quad x = 3$



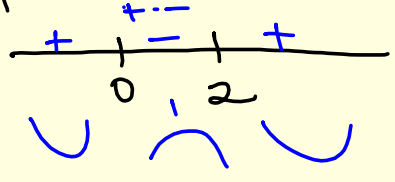
Inc $(3, \infty)$

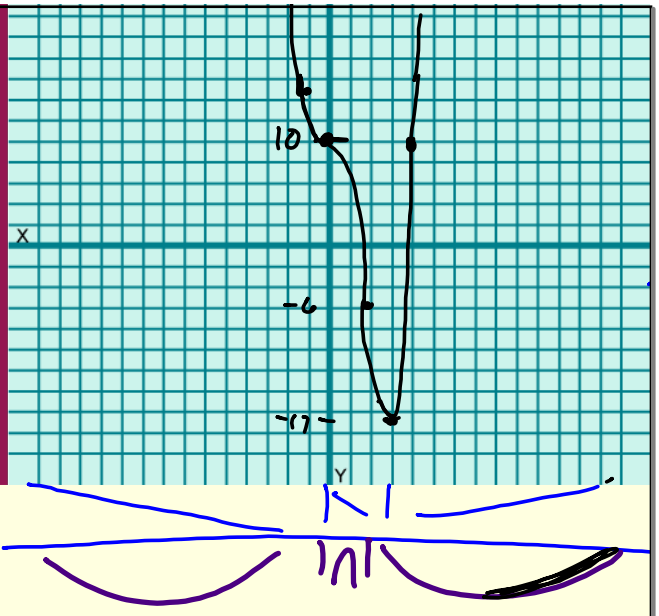
Dec $(-\infty, 0)$ $(0, 3)$

$f''(x) = 12x^2 - 24x = 0$

$12x(x-2) = 0$

Possible Inft. pts. $\rightarrow$ $x = 0 \quad x = 2$





0	10
2	-6
3	-17
-1	15
4	10

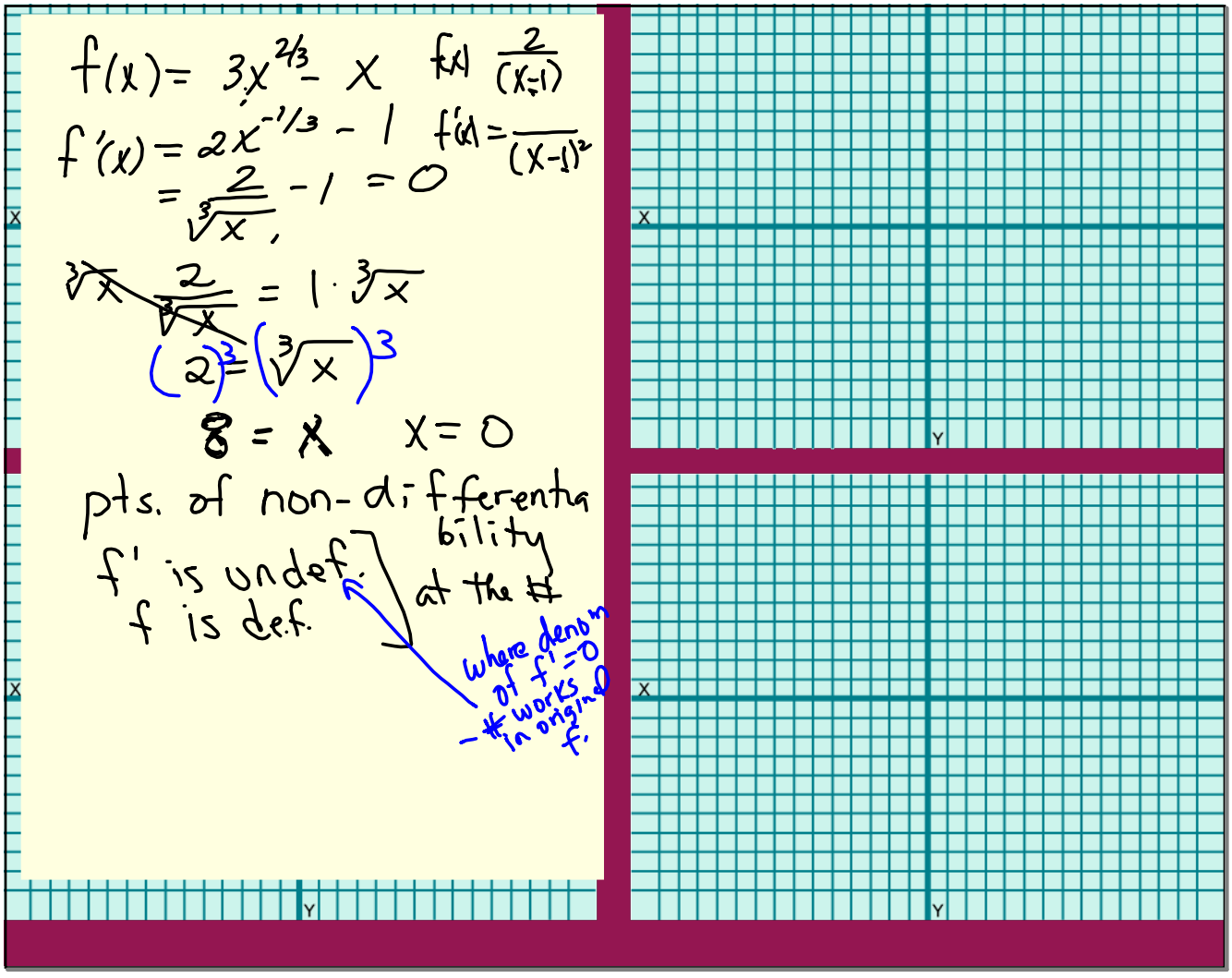
Rel max none

Rel min $(3, -17)$

Abs min $(-3, -17)$

Abs max None

Infl. pts. $(0, 10)$ $(2, -6)$


$f(x) = 3x^{2/3} - x$
$f'(x) = 2x^{-1/3} - 1$
$= \frac{2}{\sqrt[3]{x}} - 1 = 0$
$\frac{2}{\sqrt[3]{x}} = 1 \cdot \sqrt[3]{x}$
$(2)^3 = (\sqrt[3]{x})^3$
$8 = x \quad x = 0$
pts. of non-differentiability
f' is undef.
f is def.
at the #
where denom of f' = 0
works in original f.
$f(x) = \frac{2}{(x-1)}$
$f'(x) = \frac{\quad}{(x-1)^2}$
X
Y