

# ASYMPTOTES

Vertical  $\lim_{x \rightarrow \#} f(x) = \infty$

Horizontal  $\lim_{x \rightarrow \infty} f(x) = \#$

Slant - Numerator is one power higher than denominator.

Curvilinear  $2 \cdot \frac{-5}{2} = -5$   
Numerator is two or more powers higher than the denom.



$$f(x) = \frac{4x^3 - 3x^2 - 5}{2x^2 + x + 3}$$

$$\lim_{x \rightarrow \infty} \frac{4x^3}{2x^2}$$

$$\lim_{x \rightarrow \infty} 2x = \infty$$

Long Division

$$\begin{array}{r} 2x - 5/2 \\ \hline 2x^2 + x + 3 \overline{) 4x^3 - 3x^2 + 0x - 5} \\ \underline{-4x^3 + 2x^2 + 6x} \phantom{-5} \\ -5x^2 - 6x - 5 \end{array}$$

$$y = 2x - \frac{5}{2}$$

$$\lim_{x \rightarrow +\infty} f(x) = 3 \leftarrow \text{Horiz. asymp}$$

$$\lim_{x \rightarrow -2^-} f(x) = -\infty \quad \text{Vert. asymp at } x = -2$$

$$\lim_{x \rightarrow -2^+} f(x) = +\infty$$

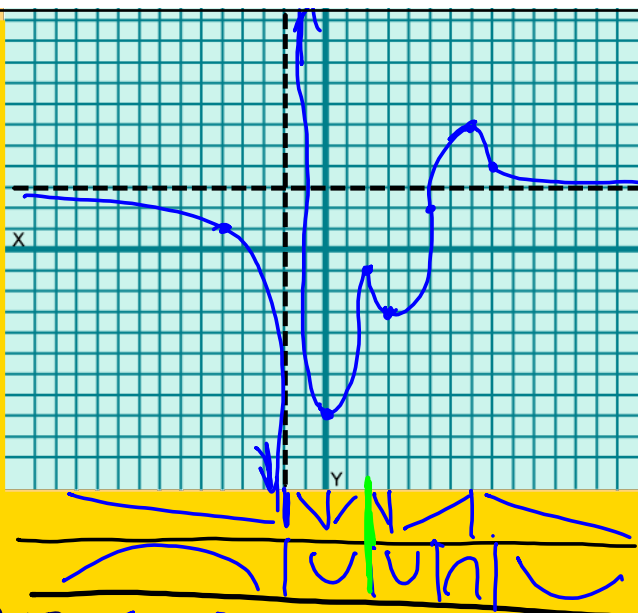
$$\begin{aligned} f(-5) &= 1 & f(0) &= -8 & f(2) &= -1 \\ f(3) &= -3 & f(5) &= 2 & f(7) &= 6 \\ f(8) &= 4 \end{aligned}$$

$$(-\infty, -2) \quad (-2, 0) \quad (2, 3) \quad (7, \infty) \quad f'(x) < 0 \leftarrow \text{Dec}$$

$$(0, 2) \quad (3, 7) \quad f'(x) > 0 \leftarrow \text{Inc}$$

$$(-\infty, -2) \quad (5, 8) \quad f''(x) < 0$$

$$(-2, 2) \quad (2, 5) \quad (8, \infty) \quad f''(x) > 0$$



Critical points

$$f'(x) = 0$$

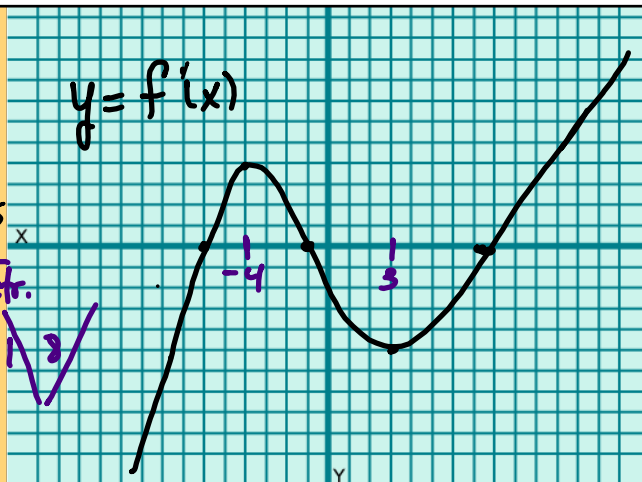
$$-6, -1, 8$$

$$\begin{array}{c} - & + & - & + \\ | & | & | & | \\ -6 & -1 & 8 & \end{array}$$

Inc  $(-6, -1)$   $(8, \infty)$ Dec  $(-\infty, -6)$   $(-1, 8)$ Above/  
below  
x-axis

Rel. Extr.

$$\begin{array}{c} \swarrow \searrow \\ -6 \quad -1 \quad 8 \\ \swarrow \searrow \end{array}$$

 $y = f'(x)$ Infl pts. of orig. — where  $f'$  at rel max/min

Infl:  $x = -4, 3$

$$\begin{array}{c} - & + & - \\ | & | & | \\ -4 & 3 & \end{array}$$

Concavity — where  $f'$  is  
inc/dec.Concave up  $(-4, 3)$ Concave down  $(-\infty, -4)$   $(3, \infty)$

