PRECALCULUS HANDOUT
PROBABILITY

Solve each problem.

1. A canister contains 20 pieces of candy: 5 grape flavored, 9 lemon flavored, and 6 cherry flavored. Four are selected at random. (a) Find the probability that two lemon candies and two cherry candies are selected. (b) Find the probability that at least one is grape.

2. A clothing store having a special anniversary sale offers customers the opportunity to draw a percentage off ticket for each item purchased. The jar of tickets contain one 50% off ticket, four 25% off tickets, six 15% off tickets, and nine 10% off tickets. (a) If Jessica purchases 3 items, what are the odds that she selects two 10% off tickets and one 25% off ticket? (b) What is the probability that she selects no more than two 15% off tickets?

3. In order to make a cook's choice lunch more exciting, the NCHS cooks have hidden the three entrees being served and are asking students to randomly select from one of three pans to determine which entrée they get. By the time the last four students come through the line, there are 5 spicy chickens, 4 BBQ ribs, and 2 chicken fried steaks remaining. (a) What is the probability that the last four students select 3 spicy chickens and 1 chicken fried steak? (b) What is the probability that they select at most 3 BBQ ribs?

4. A cookie jar contains 4 sugar cookies, 6 chocolate chip cookies, and 3 oatmeal raisin cookies. (a) If Horace selects 3 cookies and eats them. What is the probability that he selects a sugar cookie, a chocolate chip cookie, and then another sugar cookie? (b) What are the odds that he selects an oatmeal cookie, puts it back, and then selects a chocolate chip cookie?

5. As a fundraiser StuCo allows students to draw 4 colored squares out of a jar for \$1. If they get the same color on all 4 draws, their names go in a hat to win \$100. The jar contains 6 purple squares, 5 white squares, and 4 blue squares. Squares are not replaced after each draw. (a) If Janice pays \$1, what are the odds that she selects the squares in the order purple, blue, purple, blue? (b) What are the odds that she selects 4 purple squares?

6. A storage drawer in Ken's refrigerator contains 4 oranges, 3 apples, and 5 bananas. Ken randomly grabs a piece of fruit out of the drawer when he is hungry. What is the probability that the next 3 pieces of fruit Ken eats will be a banana, then an orange, followed by another banana?

7. A bag contains 15 billiard balls, number consecutively from 1 to 15. Four balls are selected at random. What is the probability that 4 odd-numbered balls or 4 balls under 10 are selected?

8. Ten students, 4 boys and 6 girls, are sitting on the stage steps. Three of the boys and two of the girls have black book bags. One of the boys and four of the girls have blue book bags. If three students are randomly selected, what is the probability of selecting three girls or three students with blue book bags?

9. Marlene's jewelry box contains 12 pair of earrings. Seven pairs of earrings are silver while five pairs are gold. Four of the silver earrings are hoops while the other three are posts. Two of the gold pairs are hoops while the remaining three pair are posts. If Marlene randomly selects three pairs of earrings, what is the probability that she selects three hoops or three silver earrings?

10. A space vehicle has an independent backup system for one of its communication networks. The probability that either system will function satisfactorily for the duration of a flight is 0.985. What is the probability that during a given flight (a) both systems fail? (b) at least one system functions satisfactorily?

ANSWERS

1. (a) $\frac{36}{323}$ (b) $\frac{232}{323}$
2. (a) $\frac{12}{83}$ (b) $\frac{56}{57}$
3. (a) $\frac{2}{33}$ (b) $\frac{329}{330}$
4. (a) $\frac{6}{143}$ (b) $\frac{18}{151}$
5. (a) $\frac{1}{90}$ (b) $\frac{1}{90}$
6. $\frac{2}{33}$
7. $\frac{191}{1365}$
8. $\frac{13}{60}$
9. $\frac{51}{220}$
10. a) 0.0002 b) 0.9998

Binomial Probability

Answers

5. 0.116
6. 0.391
9. a) $\frac{1701}{2048}$
b) $\frac{9}{2048}$
11. a) $\frac{64}{125}$
b) $\frac{1}{125}$
c) $\frac{124}{125}$
12. a) $\frac{128}{2187}$
c) $\frac{448}{2187}$ d) $\frac{1808}{2187}$
15. a) 0.396
b) No; her free throw % changes each time she shoots.
17. a) 0.387
18. 0.263
17. b) 0.987

WRITTEN EXERCISES

A

1. Make a table, like the ones on page 613, showing the 8 different ways in which "heads" (H) and "tails" (T) can occur if a coin is tossed 3 times. Then find the probability of getting:
   a. 3 "heads" b. 2 "heads" c. 1 "head" d. 0 "heads"
2. Make a table, like the ones on page 613, showing the 16 different ways in which sixes (S) and non-sixes (N) can occur when a die is rolled 4 times. Then find the probability of getting:
   a. 4 sixes b. 3 sixes c. 2 sixes d. 1 six e. 0 sixes
3. Consider the set of families with exactly 4 children. If $P(\text{child is a boy}) = \frac{1}{2}$, find the probability that one of these families, picked at random, has:
   a. 4 boys b. 3 boys c. 2 boys d. 1 boy e. 0 boys $\frac{1}{16}$
4. Suppose a coin is bent so that the probability of its coming up "heads" on any toss is $\frac{2}{5}$. If the coin is tossed 3 times, find the probability of getting:
   a. 3 "heads" b. 2 "heads" c. 1 "head" d. 0 "heads" $\frac{27}{..}$
5. What is the probability of getting exactly 2 fives in 4 rolls of a die?
6. What is the probability of getting exactly 1 three in 7 rolls of a die?
7. a. If one card is drawn from a well-shuffled standard deck, what is the probability of drawing a spade?
   b. If one card is drawn from each of two well-shuffled standard decks, what is the probability of drawing 0 spades? 1 spade? 2 spades?
8. If one card is drawn from each of three well-shuffled standard decks, find the probability of drawing:
   a. 3 spades b. 2 spades c. 1 spade d. 0 spades
9. A quiz has 6 multiple-choice questions, each with 4 choices. If you guess at every question, what is the probability of getting:
   a. no more than 2 questions right? b. 5 out of 6 questions right? $\frac{9}{8}$
10. A jar contains 4 red balls and 3 white balls, all the same size. Suppose you pull out a ball and note its color, put it back, and mix up the contents of the jar. If you do this twice more, find the probability of getting:
    a. 0 red balls b. 1 red ball c. 2 red balls d. 3 red balls

B

11. Eight out of every ten nutritionists recommend Brand X. If nutritionists A, B, and C are asked their opinions on Brand X, what is the probability that:
    a. all three recommend Brand X? b. none recommends Brand X? $\frac{1}{25}$
    c. at least one recommends Brand X?
12. In a certain high school, one third of the senior boys are at least 6 ft tall. In a randomly selected group of 7 senior boys, what is the probability that:
    a. all are less than 6 ft tall? b. none are less than 6 ft tall?
    c. all but one are less than 6 ft tall? d. no more than 3 are 6 ft. tall?

15. a. **Sports** A basketball player's free-throw percent is .750. What is the probability that she scores on exactly 4 of her next 5 free throws?
    b. *Discussion* Using the binomial probability theorem to complete part (a) assumes that the probability of a successful free throw is always .750. Is this assumption valid? Explain.

17. In a package of tomato seeds, 9 seeds out of 10 sprout on the average. a) What is the probability that of the first 10 seeds planted, 1 does not sprout? b) What is the probability that at least 7 sprout?
18. One out of every 5 boxes of Rice Toasties has a secret message decoder ring. You buy 5 boxes hoping to get at least 2 rings. What are your chances?

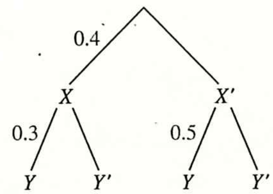
CONDITIONAL PROBABILITY

WRITTEN EXERCISES

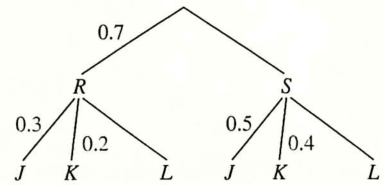
A

1. Use the tree diagram at the left below to find each probability.

a. $P(X')$ **b.** $P(Y' \mid X)$ **c.** $P(Y' \mid X')$ **d.** $P(X \text{ and } Y)$
e. $P(X' \text{ and } Y)$ **f.** $P(Y)$ **g.** $P(X \mid Y)$ **h.** $P(X' \mid Y)$



Ex. 1



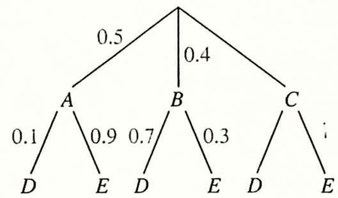
Ex. 2

2. Use the tree diagram at the right above to find each probability.

a. $P(S)$ **b.** $P(L \mid R)$ **c.** $P(L \mid S)$ **d.** $P(J \text{ and } R)$
e. $P(J \text{ and } S)$ **f.** $P(J)$ **g.** $P(R \mid J)$ **h.** $P(S \mid J)$

3. Use the tree diagram at the right to find each sum of probabilities.

a. $P(A \text{ and } D) + P(A \text{ and } E)$
b. $P(B \text{ and } D) + P(B \text{ and } E)$
c. $P(C \text{ and } D) + P(C \text{ and } E)$



Exs. 3, 4

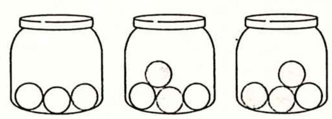
4. Refer to the tree in Exercise 3 and suppose that event D cannot possibly happen if event C happens. Find $P(A \mid D)$, $P(B \mid D)$, and $P(C \mid D)$.

5. Jar A contains 2 red balls and 3 white balls. Jar B contains 4 red balls and 1 white ball. A coin is tossed. If it shows "heads," a ball is randomly picked from Jar A; if it shows "tails," a ball is randomly picked from Jar B.

A B

a. Draw a tree diagram showing the probabilities of each jar and then the probabilities of picking a red ball or a white ball.
b. Find the probability of picking a red ball.
c. If a red ball is picked, find the probability that it came from Jar A.

6. Jars A, B, and C contain red and white balls as shown. A die is rolled. If an even number comes up, a ball is randomly picked from Jar A. If a "1" or a "3" comes up, a ball is randomly picked from Jar B. If a "5" comes up, a ball is randomly picked from Jar C.



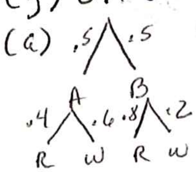
A B C

a. Incorporate the facts given above into a tree diagram.
b. Find the probability of picking a red ball.
c. If a red ball is picked, what is the probability that it came from Jar A? from Jar B? from Jar C?

1 (a) 0.6
(b) 0.7
(c) 0.5
(d) 0.12
(f) 0.42
(g) 0.286

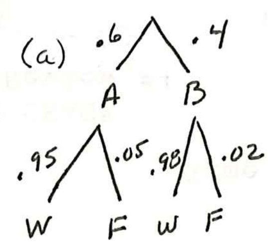
2 (c) 0.1
(e) 0.15
(f) 0.36
(g) 0.583

5. (a)



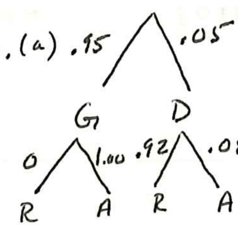
(b) .6
(c) .333

7. (a)



(b) 3.8%
(c) 0.789

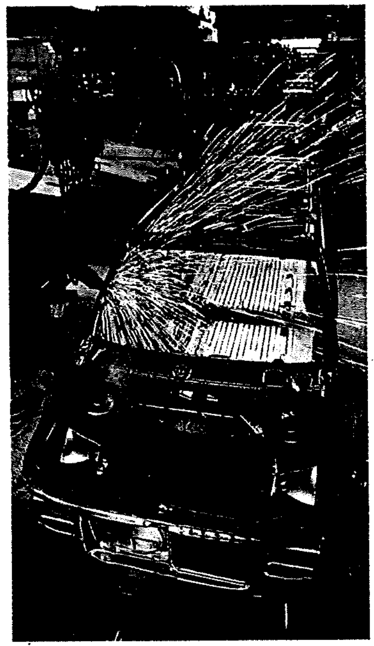
8. (a)



(b) 95.4%
(c) 0.004

7. **Manufacturing** Machine A produces 60% of the ball bearings manufactured by a factory and Machine B produces the rest. Five percent of Machine A's bearings fail to have the required precision, and two percent of Machine B's bearings fail.

a. Incorporate the facts given above into a tree diagram.
b. What percent of the bearings fail to have the required precision?
c. If a bearing is inspected and fails to have the required precision, what is the probability that it was produced by Machine A?



8. **Manufacturing** Five percent of the welds on an automobile assembly line are defective. The defective welds are found using an X-ray machine. The machine correctly rejects 92% of the defective welds and correctly accepts all of the good welds.

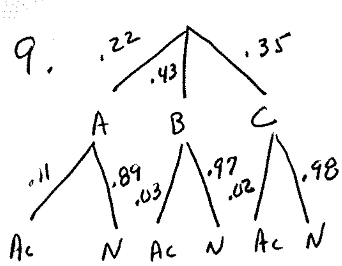
a. Incorporate the facts given above into a tree diagram.
b. What percent of the welds are accepted by the machine?
c. Find the probability that an accepted weld is defective.

9. **Insurance** An auto insurance company charges younger drivers a higher premium than it does older drivers because younger drivers as a group tend to have more accidents. The company has 3 age groups: Group A includes those under 25 years old, 22% of all its policyholders. Group B includes those 25–39 years old, 43% of all of its policyholders. Group C includes those 40 years old or older. Company records show that in any given one-year period, 11% of its Group A policyholders have an accident. The percentages for groups B and C are 3% and 2%, respectively.

a. What percent of the company's policyholders are expected to have an accident during the next 12 months?
b. Suppose Mr. X has just had a car accident. If he is one of the company's policyholders, what is the probability that he is under 25?

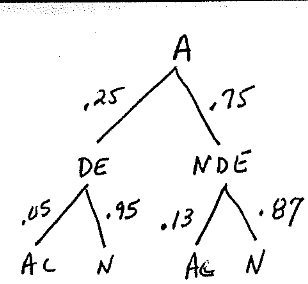
10. **Insurance** Suppose the insurance company of Exercise 9 not only classifies drivers by age, but in the case of drivers under 25 years old, it also notes whether they have had a driver's education course. One quarter of its policyholders under 25 have had driver's education and 5% of these have an accident in a one-year period. Of those under 25 who have not had driver's education, 13% have an accident within a one year period. A 20-year-old woman takes out a policy with this company and within one year she has an accident. What is the probability that she did *not* have a driver's education course?

9.



(a) 4.41%
(b) 0.549

10.



(a) 0.886

B 11. **Medicine** A medical research lab proposes a screening test for a disease. In order to try out this test, it is given to 100 people, 60 of whom are known to have the disease and 40 of whom are known not to have the disease. A positive test indicates the disease and a negative test indicates no disease. Unfortunately, such medical tests can produce two kinds of errors:

(1) A *false negative* test: For the 60 people who do have the disease, this screening test indicates that 2 do *not* have it.

(2) A *false positive* test: For the 40 people who do not have the disease, this screening test indicates that 10 *do* have it.

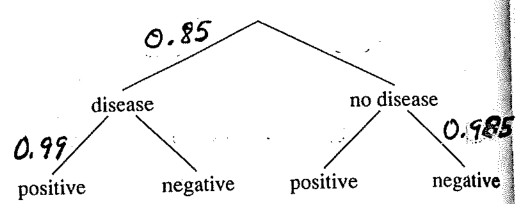
a. Which of the false tests do you think is more serious? Why?

b. Incorporate the facts given above into a tree diagram. (Be sure to convert the given integers into probabilities.)

c. Suppose the test is given to a person not in the original group of 100 people. It is not known whether this person has the disease, but the test result is positive. What is the probability that the person really does have the disease?

d. Suppose the test is given to a person whose disease status is unknown. If the test result is negative, what is the probability that the person does *not* have the disease?

12. **Medicine** Part (c) of Exercise 11 indicates that about 85% of those who test positive really do have the disease, so that 15% of those who test positive do not have it. This 15% error may seem high, but people with a positive screening test are usually given a more thorough diagnostic test. Even the diagnostic test can yield errors but they are much less likely than the screening test, as the diagram shows.



a. What is the probability that the diagnostic test gives:

(1) a false negative result?

(2) a false positive result?

b. What is the probability that:

(1) the diagnostic test gives the correct result?

(2) a person with a positive diagnostic test has the disease?

(3) a person with a negative diagnostic test does not have the disease?

13. The children of a math professor play two games that use dice. In one game, two dice are rolled and the sum of the numbers on the dice is called out. In the other game, a single die is rolled and its number is called out. The professor hears the children in another room call out the number 2, and knowing that they play the two games about equally often, the professor is able to calculate the probability they are playing the two-dice game. What is this probability?

14. Solve Exercise 13 if the children call out:

a. the number 4 $\frac{1}{3}$ **b.** the number 7 1 **c.** the number 1 0

12. a) 1) 0.0085
2) 0.00225
b) 1) 0.989
2) 0.997
3) 0.946

EXPECTED VALUE

5. Suppose you toss 3 coins and win payoffs as shown in the table below. Complete the table and find your expected payoff.

Number of "heads"	3	2	1	0
Payoff	\$5	\$3	\$1	−\$9
Probability	?	?	?	?

$\frac{1}{8}$; $\frac{3}{8}$; $\frac{3}{8}$; $\frac{1}{8}$; \$1

6. *Discussion* Discuss why the student in Example 3 may want the car collision damage insurance even though its expected value to the student is negative.

WRITTEN EXERCISES

Find the expected payoff.

A

1.

Payoff	9	7	−5
Probability	0.1	0.3	0.6

2.

Payoff	6	3	−5
Probability	0.2	0.1	0.7

3.

Payoff	60	52	50
Probability	0.4	0.5	0.1

4.

Payoff	13	−7	−12
Probability	0.4	0.2	0.4

For Exercises 5–8, decide if each game is a fair game. If not, state which player has the advantage.

5. A die is rolled. If the number that shows is odd, player A wins \$1 from player B. If it is a 6, A wins \$2 from B. Otherwise B wins \$3 from A.
6. A box contains 2 red balls and 1 white ball. Two balls are randomly chosen without replacement. If both are red, player A wins \$5 from player B. Otherwise B wins \$2 from A.
7. Two dice are rolled. If the sum is 6, 7, or 8, player A wins \$5 from player B. Otherwise B wins \$4 from A.
8. Two dice are rolled. If the sum of the numbers showing on the dice is odd, player A wins \$1 from player B. If both dice show the same number, A wins \$3 from B. Otherwise B wins \$3 from A.
9. Suppose you play a game in which you make a bet and then draw a card from a well-shuffled deck that includes the standard 52 cards as well as 2 jokers. If you draw a joker, you keep your bet and win \$5; if you draw a face card, you keep your bet and win \$2; and if you draw any other card, you lose your bet. What is your expected gain or loss on this game if your bet is \$1?
10. Suppose you have \$10 to bet on the game described in Exercise 9. Is your expected gain or loss any different if you bet the whole \$10 on one game rather than betting \$1 at a time on 10 successive games?

2. −2
4. −1
5. Not fair; B
6. Not fair; A
9. An 11¢ loss
10. one \$10 game = −\$6.78
10 \$1 games = −\$1.10
11. 0
13. 50¢ gain
17. \$20,850
19. + 1200
20. −18.20
21. 15¢ loss
22. −21¢
18. 95.5¢
95,500

B 11. **Test Taking** On a multiple-choice test, a student is given five possible answers for each question. The student receives 1 point for a correct answer and loses $\frac{1}{4}$ point for an incorrect answer. If the student has no idea of the correct answer for a particular question and merely guesses, what is the student's expected gain or loss on the question?

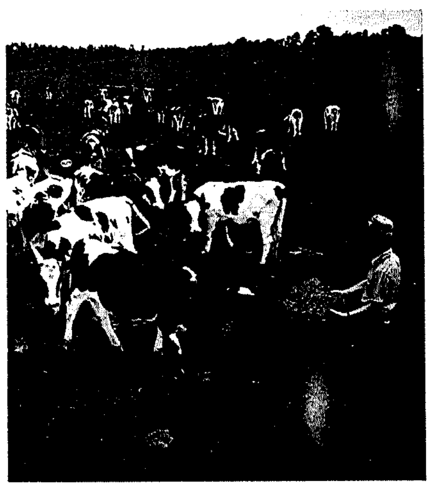
12. **Test Taking** Suppose you are taking the multiple-choice test described in Exercise 11. Suppose also that on one of the questions you can eliminate two of the five answers as being wrong. If you guess at one of the remaining three answers, what is your expected gain or loss on the question?

13. A box contains 3 red balls and 2 green balls. Two balls are randomly chosen without replacement. If both are green, you win \$2. If just one is green, you win \$1. Otherwise you lose \$1. What is your expected gain or loss?

14. In the carnival game "chuck-a-luck," you pick a number from 1 to 6 and roll 3 dice in succession. If your number comes up all 3 times, you win \$3; if your number comes up twice, you win \$2; if it comes up once, you win \$1; otherwise you lose \$1. What is your expected gain or loss? (*Hint*: Make a table like the one in Class Exercise 5.)

15. Rewrite the definition of expected value (page 630) using sigma notation.

16. Rewrite the definition of expected value (page 630) as a dot product of two vectors.



17. **Farming** A dairy farmer estimates that next year the farm's cows will produce about 25,000 gallons of milk. Because of variation in the market price of milk and the cost of feeding the cows, the profit per gallon may vary with the probabilities given in the table below. Estimate the profit on the 25,000 gallons.

Gain per gallon	\$1.10	\$.90	\$.70	\$.40	\$.00	−\$.10
Probability	0.30	0.38	0.20	0.06	0.04	0.02

18. **Insurance** At many airports, a person can pay only \$1 for a \$100,000 life insurance policy covering the duration of the flight. In other words, the insurance company pays \$100,000 if the insured person dies from a possible flight crash; otherwise the company gains \$1 (before expenses). Suppose that past records indicate 0.45 deaths per million passengers. How much can the company expect to gain on one policy? on 100,000 policies?

19. **Business** A construction company wants to submit a bid for remodeling a school. The research and planning needed to make the bid cost \$4000. If the bid is accepted, the company would make \$26,000. Would you advise the company to spend the \$4000 if the bid has only a 20% probability of being accepted? Explain your reasoning.

20. **Consumer Economics** Suppose the warranty period for your family's new television is about to expire and you are debating about whether to buy a one-year maintenance contract for \$35. If you buy the contract, all repairs for one year are free. Consumer information shows that 12% of the televisions like yours require an annual repair that costs \$140 on the average. Would you advise buying the maintenance contract? Explain your reasoning.

21. A lottery has one \$1000 prize, five \$100 prizes, and twenty \$10 prizes. What is the expected gain from buying one of the 2000 tickets sold for \$1 each?

22. In a state lottery, five numbers are randomly chosen from the numbers 1 to 30. If you pick all 5 numbers, you win \$100,000; and if you pick 4 of the 5 numbers, you win \$100. What is the value of a \$1 lottery ticket?

23. Players A and B are playing a game in which A wins a point every time a coin lands "heads" and B wins a point every time the coin lands "tails." (No points are lost by either player.) The first person to reach 3 points wins \$100. If A currently has 2 points and B has 1 point, what is A's expected gain? (*Hint*: Make a tree diagram showing the ways in which the game can be finished. From the diagram, determine the probability that A wins.)

24. Suppose you play a game in which you make a bet, toss a coin, and either win an amount equal to your bet or lose your bet depending on whether you correctly call "heads" or "tails." Also suppose you begin with a \$1 bet and double your bet on each toss until you win once and leave the game or until you have lost \$15. What can you expect to win with this betting strategy?

C 25. A die is rolled repeatedly until a "1" appears.

a. Complete the table.

Number of rolls until "1" appears	1	2	3	4	5	⋯	n	⋯
Probability	$\frac{1}{6}$	$\frac{5}{6}\cdot\frac{1}{6}$	$\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}$	?	?	⋯	?	⋯

b. Express the expected number of rolls as the sum of an infinite series. Then factor $\frac{1}{6}$ from the sum.

c. It can be proved (most easily with calculus) that

$$\sum_{n=1}^{\infty} nx^{n-1} = \frac{1}{(1-x)^2}.$$

Use this to show that the expected number of rolls until a "1" appears is 6.