December 7, 2021

SEMESTER REVIEW

LIMIT TREE

- $x \to \#$: Sub # in
  - $\frac{\infty}{\infty}, \frac{0}{0}$
    - factor
    - conjugate
    - L'Hopital's $\frac{f'(x)}{g'(x)}$
  - $\#/0$
    - Check one-sided limits for $\pm\infty$
- $x \to \pm\infty$
  - Rational: Use the highest power terms or L'Hopital's
  - Polyn.: Use highest power term
- Special Cases
  - $\lim_{x\to 0} \frac{\sin x}{x} = 1$
  - $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

$$\lim_{x\to-\infty} \frac{\sqrt{3+16x^2}}{5-2x}$$

$$\lim_{x\to-\infty} \frac{\sqrt{16x^2}}{-2x}$$

$$\lim_{x\to-\infty} \frac{4|x|}{-2x}$$

$$\lim_{x\to-\infty} \frac{-4\cancel{x}}{-2\cancel{x}} = \boxed{2}$$

L'Hopitals

$$\lim_{x\to-\infty} \frac{\cancel{\tfrac{1}{2}}(3+16x^2)^{-1/2}\cdot \cancel{32x}\ \cancel{16x}\ -8x}{\cancel{-2}}$$

$$\lim_{x\to-\infty} \frac{-8x}{\sqrt{3+16x^2}} = \frac{\infty}{\infty}$$

$$\lim_{x\to-\infty} \frac{\cancel{-8}}{\cancel{\tfrac{1}{2}}(3+16x^2)^{-1/2}\cdot \cancel{32x}\ \cancel{16x}\ -2x}$$

$$\lim_{x\to-\infty} \frac{\sqrt{3+16x^2}}{-2x}$$

L'Hopitals

$0\cdot\infty,\ \infty-\infty,\ 0^0,\ 1^\infty,\ \infty^0$

$$\lim_{x\to0^+} (\cos x)^{\frac{1}{x}} = 1^\infty$$

$$\lim_{x\to0^+} e^{\ln(\cos x)^{1/x}}$$

$$\lim_{x\to0^+} \frac{\ln(\cos x)}{x} = \frac{0}{0}$$

$$\lim_{x\to0^+} \frac{\frac{1}{\cos x}\cdot -\sin x}{1}$$

$$\lim_{x\to0^+} -\frac{\sin x}{\cos x} = \frac{0}{1} = 0$$

$$e^0 = 1$$

$$\lim_{x\to0} (\sin x)^{\tan x}$$

$$\lim_{x\to0} \tan x \cdot \ln(\sin x)$$

$$\lim_{x\to0} \frac{\ln(\sin x)}{\cot x}$$

Guaranteed

1) What does a derivative represent?

2) Write two def. of derivative:

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} \qquad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

* 6 derivatives of trig func.

$$\frac{d}{dx} e^x = e^x \qquad \frac{d}{dx} a^x = \ln a \cdot a^x$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \tan^{-1} x = \frac{1}{x^2 + 1}$$

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2 - 1}}$$

Find $f'(x)$. $f(x) = x^{3x^2}$

$$= e^{\ln x^{3x^2}} = e^{3x^2 \cdot \ln x}$$

$$f'(x) = e^{3x^2 \cdot \ln x} \cdot \left[3x^{\cancel{2}\,1} \cdot \cancel{\tfrac{1}{x}} + \ln x \cdot 6x\right]$$

$$= 3x^1 \cdot x^{3x^2} \left[1 + 2\ln x\right]$$

$$= 3x^{3x^2+1} \left[1 + 2\ln x\right.$$