

CURVE SKETCHING REVIEW

Test around asymptotes!

2/ Inc / Dec
 1) crit pts.
 2) Test pts.

3/ Concave Up/Down
 2) $f'' = 0$
 2) Test pts.

Infl pts $\frac{+}{-2} \frac{-}{0} \frac{-}{2} \frac{+}{}$
 \downarrow
 $x = -2, 2$

Relative Extrema



1st Deriv Test

- 1) find crit pts
- 2) Test pts
- 3) Mountain Test

2nd Deriv Test

- 1) Find crit pts.
- 2) Put crit pts in f'' .

$$f''(-3) = + \text{ or } -$$

\swarrow \searrow
 Rel min Rel max

Pts of non-diff.

$f'(x) = \text{no fraction}$

$f'(x) = \text{fraction} \leftarrow \text{where denom} = 0$

$$\left. \begin{array}{l} f(x) = x^{2/3} \\ f'(x) = \frac{2}{3}x^{-1/3} \\ = \frac{2}{3x^{1/3}} \\ x=0 \end{array} \right\}$$

6/ Absolute Extrema — Most missed!

[#, #]

$(-1, 8)$

1) find crit

$$f(x) = x^3 - 6x^2$$

64-96

T

$$\lim_{x \rightarrow -1} x^3 - 6x^2 = -1 - 6 = -7$$

$$\lim_{x \rightarrow 8} x^3 - 6x^2 = 512 - 384 = \underline{128}$$

$$f'(x) = 3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$$x = 0, 4$$

$$\begin{array}{r|l} 0 & 0 \\ 4 & -32 \end{array}$$

Abs min $(4, -32)$
No Abs max

Vertical

$$\lim_{x \rightarrow \#} f(x) = \pm \infty$$

Horiz

$$\lim_{x \rightarrow \pm \infty} f(x) = \#$$

Slants/Curvilinear - Long Division

#9 Graph of Deriv.

Crit pts = -3, 3

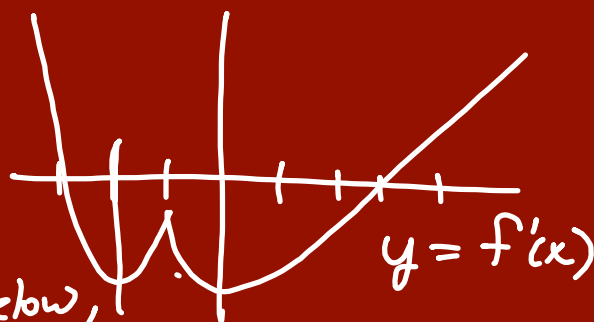
$f'(x) = 0 = x - 12x$

Inc/Dec - where $f'(x) = +/ -$
above/below

$$\begin{array}{c} + \quad - \quad + \\ \hline -3 \quad 3 \end{array}$$

Inc $(-\infty, -3) (3, \infty)$

Dec $(-3, 3)$



Concavity = $f''(x)$ has \pm and
where f' has \pm
Inc/Dec

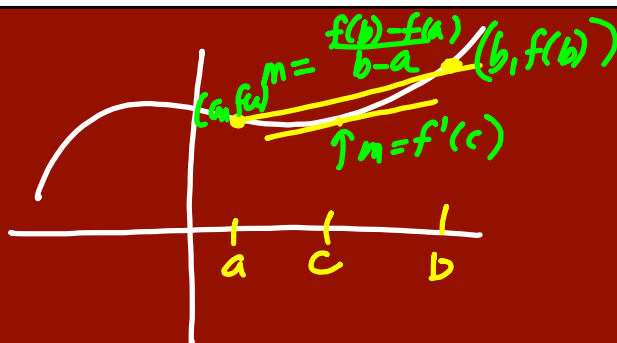
$$\begin{array}{c} - \quad + \quad - \quad + \\ \hline -2 \quad -1 \quad 0 \end{array}$$

Concave Up $(-2, -1) (0, \infty)$

Concave Down $(-\infty, -2) (-1, 0)$

Mean Value Thm

- 1) f continuous $[a, b]$
- 2) f differentiable (a, b)
- 3) $f'(c) = \frac{f(b) - f(a)}{b - a}$



1) (a) $f(x) = \frac{1}{3}x^3 + 2x$ $(0, 3)$

$$f'(x) = x^2 + 2$$

$$c^2 + 2 = \frac{f(3) - f(0)}{3 - 0}$$

$$c^2 + 2 = \frac{15 - 0}{3 - 0}$$

$$c^2 + 2 = 5$$

$$\sqrt{c^2} = \sqrt{3}$$

$$c = \pm \sqrt{3}$$

$$\approx \pm 1.732$$

$(0, 3)$

not in interval

