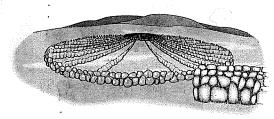
TRIGONOMETRY HANDOUT Arc Length, Area of a Sector, Angular & Linear Velocity

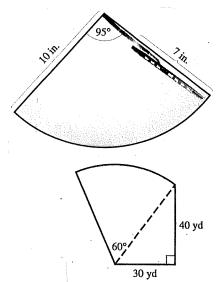
Arc Length & Area of a Sector

- 1. Find the distance in kilometers between each pair of cities, assuming they lie on the same line of longitude.
 - a) Panama City, Panama, 9° N and Pittsburgh, Pennsylvania, 40° N
 - b) New York City, New York, 41° N and Lima, Peru, 12° S
- 2. Madison, South Dakota, and Dallas, Texas, are 1200 km apart and lie on the same line of longitude. The latitude of Dallas is 33° N. What is the latitude of Madison?
- 3. A railroad track in the desert is 3.5 km away. A train on the track subtends (horizontally) an angle of 3° 20'. Find the length of the train.
- 4. The full moon subtends an angle of $1/2^{\circ}$. The moon is 240,000 miles away. Find the diameter of the moon.
- 5. The figure at right shows Medicine Wheel, a Native American structure in northern Wyoming. This circular structure is perhaps 2500 years old. There are 27 aboriginal spokes in the wheel equally spaced.
 - a) Find the measure of each central angle in degrees & radians.
 - b) If the radius of the wheel is 76 feet, find the circumference.
 - c) Find the length of each arc intercepted by consecutive pairs of spokes.
 - d) Find the area of each sector formed by consecutive spokes.
- 6. The Ford Model A, built from 1928 to 1931, had a single windshield wiper on the driver's side. The total arm and blade was 10 inches long and rotated back and forth through an angle of 95°. The shaded region in the figure is the portion of the windshield cleaned by the 7-inch wiper blade. What is the area of the region cleaned?
- 7. A frequent problem in surveying city lots and rural lands adjacent to curves of highways and railways is the of finding the area when one or more the boundary lines is the arc of a circle. Find the area of the lot shown in the figure.

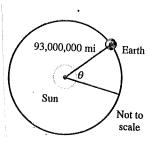


- 8. Find *v* for each of the following:
 - a) a point on the tread of a tire of radius 18 cm, rotating 35 times per minute
 - b) the tip of an airplane propeller 3 m long, rotating 500 times per minute.
- 9. The tires of a bicycle have radius 13 inches and are turning at the rate of 200 revolutions per minute. How fast is the bicycle traveling in miles per hour? (5280 ft = 1 mile)





- 10. Earth travels about the sun in an orbit that is almost circular. Assume that the orbit is a circle, with radius 93,000,000 miles. Its angular and linear velocities are used in designing solar power facilities.
 - a) Assume that a year is 365 days. Find θ , the angle formed by Earth's movement in one day.
 - b) Give the angular velocity in radians per hour.
 - c) Find the linear velocity of Earth in miles per hour.



- 11. A pulley has a radius of 12.96 cm. Suppose it takes 18 seconds for 56 cm of belt to go around the pulley. Find the angular velocity of the pulley in radians per second.
- 12. A thread is being pulled off a spool at the rate of 59.4 cm per second. Find the radius of the spool if it makes 152 revolutions per minute.
- 13. A railroad track is laid along the arc of a circle of radius 1800 feet. The circular part of the track subtends a central angle of 40°. How long (in seconds) will it take a point on the front of a train traveling 30 mph to go around this portion of the track?

ANSWERS

- 1. a) 3500 km. b) 5900 km
- 2. 44° N
- 3. 0.20 km
- 4. 2100 mi
- 5. a) $13\frac{10}{3}$, $\frac{2\pi}{27}$ b) 480 ft. c) 17.7 ft. d) 672 ft²
- 6. 75.4 in^2
- 7. 1900 vd²
- 8. a) $1260\pi = 3958.4 \approx 4000 \text{ cm/min}$ b) $1500\pi = 4712.4 \approx 5000 \text{ m/min}$
- 9. 15.5 mph
- 10. a) $\frac{2\pi}{365}$ b) $\frac{\pi}{4380} \frac{rad}{hr}$ c) 66,700 mph
- 11. 0.24 rad/sec
- 12.3.73 cm
- 13.29 sec

TRIGONOMETRY WORKSHEET Trig Basics

Identify the quadrant or quadrants for the angle satisfying the given conditions.

1.
$$\sin \alpha > 0$$
, $\cos \alpha < 0$

2.
$$\sec \beta < 0$$
, $\csc \beta < 0$

3.
$$\tan \gamma < 0$$
, $\cot \gamma < 0$

4.
$$\csc\theta < 0$$
, $\sec\theta > 0$

Determine whether each statement is possible or impossible.

5.
$$\sin \alpha = 2$$

6.
$$\sec \omega - 4 = 1$$

7.
$$8\cot\varphi - 5 = 3$$

8.
$$\sin \alpha = \frac{1}{2}$$
 and $\csc \alpha = 2$

9.
$$\tan \theta = 2$$
 and $\cot \theta = -2$

Write each of the following in terms of its cofunction (complementary function).

11.
$$\cos \frac{2\pi}{5}$$

Use the given value to evaluate the trigonometric function.

13.
$$\cos(-\theta) = \frac{8}{11}$$
; Find $\cos \theta$ and $\sec \theta$.

14.
$$\sin(-\theta) = \frac{5}{9}$$
; Find $\sin \theta$ and $\csc \theta$.

15.
$$\sec(\theta) = \frac{13}{12}$$
; Find $\cos\theta$ and $\cos(-\theta)$.

16.
$$\csc(\theta) = \frac{19}{17}$$
; Find $\sin \theta$ and $\sin(-\theta)$.

The terminal side of angle $\,\theta$ passes through the given coordinate. Find the value of the indicated trig functions.

17.
$$\left(-3\sqrt{2},3\right)$$
; $\csc\theta$ $\cos\theta$

18.
$$(4,-2\sqrt{6})$$
; $\cot\theta$ $\sin\theta$

19. Given $\cot \theta = -\frac{5}{3}$ and $\csc \theta < 0$, find $\sin \theta$.

20. Given $\cos \theta = -\frac{2\sqrt{5}}{7}$ and $\sin \theta < 0$, find $\tan \theta$.

21. Given $\csc \theta = \frac{7}{6}$ and $\tan \theta < 0$, find $\sec \theta$.

ANSWERS

11.
$$\sin \frac{\pi}{10}$$

13.
$$\frac{8}{11}$$
; $\frac{11}{8}$

14.
$$-\frac{5}{9}$$
; $-\frac{9}{5}$

15.
$$\frac{12}{13}$$
; $\frac{12}{13}$

16.
$$\frac{17}{19}$$
; $-\frac{17}{19}$

17.
$$\csc\theta = \sqrt{3}$$
; $\cos\theta = -\frac{\sqrt{6}}{3}$

18.
$$\cot \theta = -\frac{\sqrt{6}}{3}$$
; $\sin \theta = -\frac{\sqrt{15}}{5}$

19.
$$\sin \theta = -\frac{3\sqrt{34}}{34}$$

20. $\tan \theta = -\frac{\sqrt{145}}{10}$

20.
$$\tan \theta = \frac{\sqrt{145}}{10}$$

21.
$$\sec \theta = -\frac{7\sqrt{13}}{13}$$

TRIGONOMETRY WORKSHEET Special Angle Values

. Fill in the table with the correct value.

2.
$$\cos 150^{\circ} =$$
 4. $\sin (-210^{\circ}) =$

5.
$$\tan(-270^{\circ}) =$$
 6. $\csc 225^{\circ} =$ 7. $\sec 480^{\circ} =$ ____

8.
$$\csc 180^{\circ} =$$
 9. $\cot 210^{\circ} =$ 10. $\tan (-855^{\circ}) =$

11.
$$\cos \frac{5\pi}{6} =$$
 12. $\cot \frac{3\pi}{4} =$ 13. $\csc \frac{15\pi}{4} =$

14.
$$\sin\left(-\frac{\pi}{2}\right) = \underline{\qquad}$$
 15. $\tan\frac{11\pi}{6} = \underline{\qquad}$ 16. $\cos\left(-\frac{4\pi}{3}\right) = \underline{\qquad}$

17.
$$\sec 5\pi = \underline{\hspace{1cm}}$$
 18. $\csc \frac{5\pi}{3} = \underline{\hspace{1cm}}$ 19. $\cot \frac{3\pi}{2} = \underline{\hspace{1cm}}$

Evaluate each of the following expressions. 20. $\sin^2 315^\circ + \cos^2 135^\circ$

21. sin 225° cos 330° – sec 315° csc 120°

22.
$$\frac{\cos 240^{\circ} - \sin(-120^{\circ})}{\cos(-390^{\circ})\sin 570^{\circ}}$$

23.
$$\frac{\csc^2 315^\circ - \cos 120^\circ}{\sin 270^\circ \cot 495^\circ}$$

24.
$$1+\cot^2\frac{11\pi}{6}$$

25.
$$\cot^2 \frac{3\pi}{2} - \sec^2 \pi + \csc^2 \frac{3\pi}{4}$$

$$26. \frac{1-\tan^2 \frac{5\pi}{6}}{2\cot \frac{4\pi}{3}}$$

$$27. \frac{\sin\frac{23\pi}{6} - \cos\frac{5\pi}{4}}{\tan\frac{2\pi}{3}\sec 0}$$

Find all values of angle θ , when $0^{\circ} \le \theta < 360^{\circ}$, for which the following are true.

$$28. \sin \theta = -\frac{1}{2}$$

29.
$$\tan \theta = 1$$

30.
$$\sec \theta = 2$$

31.
$$\cos \theta = -\frac{\sqrt{3}}{2}$$

32
$$\cot \theta = -1$$

33.
$$\csc \theta$$
 is undefined.

Find all values of the angle θ , when $0 \le \theta < 2\pi$, for which the following are true.

34.
$$\sin \theta = -1$$

35.
$$\tan \theta = \frac{\sqrt{3}}{3}$$

36.
$$\cos \theta = 0$$

37.
$$\sec \theta = -\sqrt{2}$$

38.
$$\cot \theta = -\frac{\sqrt{3}}{3}$$

39.
$$\csc \theta = -\frac{2\sqrt{3}}{3}$$

ANSWERS

2.
$$-\frac{\sqrt{3}}{2}$$

3.
$$-\sqrt{3}$$

4.
$$\frac{1}{2}$$

- 5. Undefined
- 6. $-\sqrt{2}$
- 7. –2
- 8. Undefined
- 9. $\sqrt{3}$
- 10.1
- 11. $-\frac{\sqrt{3}}{2}$
- 12. –1
- 13. $-\sqrt{2}$
- 14. –1
- 15. $-\frac{\sqrt{3}}{3}$
- 16. $-\frac{1}{2}$
- 17. –1
- 18. $-\frac{2\sqrt{3}}{3}$
- 19.0
- 20. 1
- 21. $-\frac{11\sqrt{6}}{12}$

22.
$$\frac{2\sqrt{3}-6}{3}$$

- 23. $\frac{5}{2}$
- 24. 4
- 25. 1
- 26. $\frac{\sqrt{3}}{3}$
- 27. $\frac{\sqrt{3}-\sqrt{6}}{6}$
- 28. 210°, 330°
- 29. 45°, 225°
- 30. 60°, 300°
- 31. 150°, 210°
- 32. 135°, 315°
- 33. 0°, 180°
- 34. $\frac{3\pi}{2}$
- 35. $\frac{\pi}{6}$, $\frac{7\pi}{6}$
- 36. $\frac{\pi}{2}$, $\frac{3\pi}{2}$
- 37. $\frac{3\pi}{4}$, $\frac{5\pi}{4}$
- 38. $\frac{2\pi}{3}$, $\frac{5\pi}{3}$
- 39. $\frac{4\pi}{3}$, $\frac{5\pi}{3}$

Assignment:		11) 2
1) 3	5-7) 2	12-13) 1 ea.
2) 1	8) 2	14-20) 6 parts
3) 3	9) 2	21-23) 2
4) 3	10) 2	24) 2
i		

Name____

TRIGONOMETRY REVIEW Intro to Trigonometry

1. Find the smallest positive and negative angles that are coterminal with the given angle.

(a)
$$-210^{\circ}$$

(d)
$$-22^{\circ}$$

2. (a)
$$\cos(-\theta) = \frac{3}{7}$$
; Find $\cos \theta$ and $\sec \theta$.

(b)
$$\csc(\theta) = -\frac{12}{5}$$
; Find $\sin \theta$ and $\sin(-\theta)$.

3. Identify the quadrant or quadrants for the angles satisfying the following conditions.

(a)
$$\sec \theta > 0 \tan \theta < 0$$

(b)
$$\cos \theta < 0 \cot \theta < 0$$

(c)
$$\sin \theta < 0 \csc \theta < 0$$

(d)
$$\tan \theta > 0 \sin \theta < 0$$

4. Determine whether each of the following is possible or impossible.

(a)
$$\sec \theta = -\frac{2}{3}$$

(b)
$$4\sin\theta + 3 = 1$$

(c)
$$5 \tan \theta - 300 = 1,000,000$$

(d)
$$\sin \theta = \frac{2}{7}$$
 and $\csc \theta = -\frac{7}{2}$

5. $(\sqrt{7}, -3)$ is on the terminal side of angle θ . Find sec θ .

6.
$$\sin \theta = -\frac{2\sqrt{5}}{7}$$
 and $\tan \theta > 0$. Find $\sec \theta$.

Assignment:		11) 2
1)3	5-7) 2	12-13) 1 ea.
2) 1	8) 2	14-20) 6 parts
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6.
$$\sin \theta = -\frac{2\sqrt{5}}{7}$$
 and $\tan \theta > 0$. Find $\sec \theta$.

7. $\cot \theta = -\frac{5}{3}$ and $\csc \theta < 0$. Find $\sin \theta$.

8. Write each of the following in terms of its cofunction.

(c)
$$\tan \frac{2\pi}{9}$$

9. Evaluate each of the following using special angle values:

(a)
$$\tan^2 330^\circ + \sec^2 120^\circ - 3\sin^2 240^\circ$$

(b)
$$\sin \frac{5\pi}{4} - \cos \left(-\frac{5\pi}{2}\right) + \cot \frac{19\pi}{6}$$

(c)
$$\frac{\sin^2 \frac{5\pi}{3} + \cos^2 840^\circ}{\sin 315^\circ \cos \frac{3\pi}{4}}$$

10. Find the values of θ with $0^{\circ} \le \theta < 360^{\circ}$ for which the following are true:

(a)
$$\sin \theta = -\frac{\sqrt{3}}{2}$$

(b)
$$\cot \theta = \sqrt{3}$$

(c)
$$\sec \theta = -1$$

7. $\cot \theta = -\frac{5}{3}$ and $\csc \theta < 0$. Find $\sin \theta$.

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(b)
$$\cot \theta = \sqrt{3}$$

(c)
$$\sec \theta = -1$$

- 11. Find the values of θ with $0 \le \theta < 2\pi$ for which the following are true:
 - (a) $\csc \theta = -\frac{2\sqrt{3}}{3}$
- (b) $\cot \theta$ is undefined

(c)
$$\cos\theta = -\frac{\sqrt{2}}{2}$$

- 12. Convert the following angles from radians to degrees.
 - (a) $\frac{14\pi}{5}$

- (b) $\frac{3\pi}{8}$
- 13. Convert the following angles from degrees to radians.
 - (a) 265°

(b) 440°

Round your answers using significant digits unless otherwise indicated.

14. The center circle on the gym floor of the Pinkerton High School Pinwheels is to be divided into 12 equal pieshaped pieces and colored alternately in the school's colors of pink and white. How much area is to be painted in one pie-shaped piece if the diameter of the circle is 15 feet? Round to tenths.

- 15. (a) A circle has a circumference of 40 cm. Find the length of an arc intercepted by an angle of 150°.
- 16. Two cities on the equator have longitudes of 72° E and 35° W, respectively. Find the distance between the cities if the earth has a radius of 6400 km.
- 17. Two pulleys, one 6 in. and the other 2 feet in diameter, are connected by a belt. If the larger pulley rotates through 300°, how many degrees does the smaller pulley rotate?

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18. (a) The first Ferris wheel built was 250 feet in diameter. The spokes that connected each car to the center of the wheel formed angles of 9°. What was the distance between each car?
(b) Modern Ferris wheels are somewhat smaller. For example, a large Ferris wheel located in Japan is only 208 feet in diameter. The first Ferris wheel made one rotation in 45 seconds while the Japanese Ferris wheel makes one rotation in 25 seconds. Find the speed of the rider on both Ferris wheels. Round to tenths.
19. Find the angular velocity of a wheel with a diameter of 20 in., rotating at 150 revolutions every 3 minutes.
20. It takes Jupiter 11.64 years to complete one orbit around the sun. If Jupiter's average distance from the sun is 483,600,000 miles, find its orbital velocity in miles per second.
Solve each of the following problems. Round angles to the nearest degree and lengths to two significant digits.
21. The airline distance from Philadelphia to Syracuse is 260 miles, on a bearing of 335°. The distance from Philadelphia to Cincinnati is 510 miles, on a bearing of 245°. Find the bearing from Cincinnati to Syracuse.
22. A lighthouse keeper is watching a boat approach directly to the lighthouse. When she first begins watching the boat, the angle of depression of the boat is 16°. Later she notices the angle of depression of the boat is 35°. If the height of the lighthouse is 69 meters, find the distance traveled by the boat as it approached the lighthouse.

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23. The Acme Building and the Unifix building are separated by an alley. Joe is looking out a window 60 feet above the ground in the Acme building. He observes that the angle of elevation to the top of the Unifix building is 40°. If the Unifix Building is 150 feet tall, what is the angle of depression from Joe to the base of the Unifix building.

24. Use a calculator to evaluate each of the following:

(a)
$$\cos \frac{15\pi}{4}$$

(b) csc123°48'

(c) Convert 47.22° to degrees, minutes and seconds

TRIG REVIEW ANSWERS

2. (a)
$$\frac{3}{7}$$
; $\frac{7}{3}$ (b) $-\frac{5}{12}$; $\frac{5}{12}$

5.
$$\frac{4\sqrt{7}}{7}$$

6.
$$-\frac{7\sqrt{29}}{29}$$

7.
$$-\frac{3\sqrt{34}}{34}$$

8. (a)
$$\sec 8^{\circ}$$
 (b) $\sin 18^{\circ} 57' 13''$ (c) $\cot \frac{5\pi}{18}$

9. (a)
$$\frac{25}{12}$$
 (b) $\frac{-\sqrt{2}+2\sqrt{3}}{2}$ (c) 2

10. (a)
$$240^{\circ}$$
, 300° (b) 30° , 210° (c) 180°

11. (a)
$$\frac{4\pi}{3}$$
, $\frac{5\pi}{3}$ (b) 0, π (c) $\frac{3\pi}{4}$, $\frac{5\pi}{4}$

13. (a)
$$\frac{53\pi}{36}$$
 (b) $\frac{22\pi}{9}$

15.
$$\frac{50}{3}$$
 cm ≈ 17 cm

19.
$$100\pi$$
 rad/min.

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