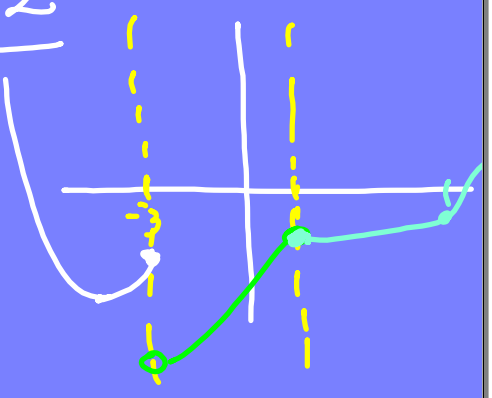
December 10, 2021

Semester Review - Day 2

$$f(x)=\begin{cases}(x+4)^2-5 & x\le -3\\ 2x-7 & -3<x<1\\ \sqrt[3]{x-9}-1 & x\ge 1\end{cases}$$



13) 1) Isolate a root.
2) Square both sides
Check solutions!

December 10, 2021

4(h) Solve $x^4 - 4x^3 - 7x^2 + 34x - 24 = 0$

$\pm 1 \quad \pm 24$
$\pm 2 \quad \pm 12$
$\pm 3 \quad \pm 8$
$\pm 4 \quad \pm 6$

$x = 1$ | 0

1 | 1 −4 −7 34 −24

$x = 1$ + 1 −3 −10 24

1 −3 −10 24 | 0

$(x-1)(x^3 - 3x^2 - 10x + 24)$

$x = 2$ | 0

2 | 1 −3 −10 24

\+ 2 −2 −24

1 −1 −12 | 0

$(x-1)(x-2)(x^2 - x - 12)$

$(x-1)(x-2)(x-4)(x+3)$

$x = 1, 2, -3, 4$

Rational Eq. / Ineq.

15) $\frac{2}{x-5} + \frac{x+3}{x-2} = 4$

Excluded values $x \neq 5, 2$

$2(x-2) + (x+3)(x-5) = 4(x-5)(x-2)$

$\frac{4}{x} < \frac{3}{x+4}$

1) Set < 0 or > 0

2) Make common denom

$0 < \frac{x \cdot 3}{x \cdot x+4} - \frac{4(x+4)}{x(x+4)}$

3) Test Points

$0 < \frac{3x - 4x - 16}{x(x+4)}$

$0 < \frac{-x-16}{x(x+4)}$

$(-\infty, -16] \cup (-4, 0)$

Decompose

16 b) $\dfrac{7x^3-8x-5}{(x^2+1)(2x^2-3)} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{2x^2-3}$

$2x^4 - x^2 - 3$

$7x^3-8x-5 = (Ax+B)(2x^2-3) + (Cx+D)(x^2+1)$

$7x^3-8x-5 = 2Ax^3-3Ax+2Bx^2-3B+Cx^3+Cx+Dx^2+D$

x^3: $7 = 2A + C$

x^2: $0 = 2B + D$

x: $-8 = -3A + C$

Constants: $-5 = -3B + D$

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ -3 & 0 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 7 \\ 0 \\ -8 \\ -5 \end{bmatrix}$$

$$\frac{4-x^2+x}{x^2(x-4)^3} = \frac{A}{(x-4)^3} + \frac{B}{(x-4)^2} + \frac{C}{x-4} + \frac{D}{x^2} + \frac{E}{x}$$

$(x+0)^2$

$$\frac{4x^2(x^2-3)^{-2/3} - 6x(x^2-3)^{1/3}}{(x^2-3)^{1/3}}$$

$\frac{1}{3}+\frac{2}{3}$

Pull out common factors!

$$\frac{2x(x^2-3)^{-2/3}\cdot[2x - 3(x^2-3)^1]}{(x^2-3)^{4/3}}$$

$+\frac{2}{3}$

$$\frac{2x[2x-3x^2+9]}{(x^2-3)^2}$$

LOGARITHMS

18/ No calculator

$9^{-3/2} = \frac{1}{\sqrt{9^3}} = \frac{1}{3^3} = \frac{1}{27}$

$\log_5 \frac{1}{125} = \log_5 \frac{1}{5^3} = \log_5 5^{-3} = -3$

$\ln e^{87} = 87$

$e^{2\ln 11} = e^{\ln 11^2} = 121$

$\log x + \log y = \log x \cdot y$

$\log x - \log y = \log \frac{x}{y}$

$\log_a x^p = p \log x$

c) $\log_7(x+9) + \log_7(x-1) = 2\log_7 6$

$\log_7(x+9)(x-1) = \log_7 6^2$

$\log_7(x^2+3x-9) = \log_7 36$

7 7

$x^2+3x-9 = 36$

$x^2+3x-45=0$

$(x-5)(x+8)=0$

$x = 5, \cancel{-8}$

Check for log of + number in original problem

Problems with logs

1) Use the properties to reduce to one log on each side
2) Exponentiate!

Like 19(h) $x^2 \cdot x^7$

$e^{2x} + 3e^x - 28 = 0$

$(e^x+7)(e^x-4)=0$

$e^x+7=0 \quad e^x-4=0$

$\ln e^x = \ln 7$ $\quad \ln e^x = \ln 4$

$x = \ln(-7)$ $\quad x = \ln 4$

Petrified frog ↑ 65% of C-14 left. Half-life of carbon-14 5700 yrs.

$q = q_0 e^{Kt}$

$0.65 = 1\, e^{-0.000124t}$

$\frac{0.5}{1} = \frac{1\, e^{K \cdot 5700}}{1}$

$\ln 0.5 = \ln e^{5700K}$

$\frac{\ln(0.5)}{5700} = \frac{5700K}{5700}$

$-0.000124 = K$

Scores on Precal Sem Exam are normally distributed with mean of 80 and st. dev. of 10.

How many students will score above 85? (22 students)

$$z = \frac{\bar{x} - \mu}{\sigma}$$

$$= \frac{85 - 80}{10} = \frac{5}{10} = 0.5$$

$0.3085 * 22 = 6.77 \approx 7$ students

Confidence Intervals – only for sampling

1) $\sigma_{\bar{x}} = \frac{s}{\sqrt{n}}$

2) $E = z \cdot \sigma_{\bar{x}}$

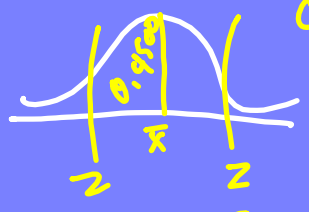
$= 1.65 * \_\_\_$

$=$

3) $\bar{x} \pm E$

90% Confident

Col A



$z = 1.65$

Hypothesis Testing

NC students $\bar{x} = 51.1$ $s = 10$

National Test $\mu = 50$ $\sigma = 10$

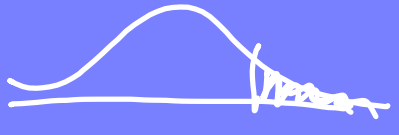
H_a: NC stud. did better $\mu = 50$ (>)

H_0: $\mu < 50$

Need: $n =$ $s =$ $\bar{x} =$

$\sigma_{\bar{x}}$

$z^* = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \_\_\_$



$p < 0.05$