December 15, 2021

Rectilinear Motion

$s(t) = 8t - t^2$ in ft. Find velocity at $t = 2$ sec.

$$V = \frac{\Delta s}{\Delta t} = \frac{s(2) - s(0)}{2 - 0} = \frac{12 - 0}{2} = 6 \frac{ft}{sec}$$

$$= \frac{s(2) - s(1)}{2 - 1} = \frac{12 - 7}{1} = 5 \frac{ft}{s}$$

$$\lim_{t \to 2^-} \frac{s(2) - s(t)}{2 - t} = v(t) = s'(t)$$

$$v(t) = 8 - 2t \quad 8 - 2(2) = \boxed{4 \frac{ft}{s}}$$

$$a(t) = \frac{\Delta v}{\Delta t} = v'(t) = s''(t)$$

$$\boxed{\begin{aligned} v(t) &= s'(t) \\ a(t) &= v'(t) = s''(t) \end{aligned}}$$

instantaneous Velocity or acceleration

$s(t) = \frac{1}{3}t^3 - 3t^2 + 8t - 6$

$v(t) = t^2 - 6t + 8$

$a(t) = 2t - 6$

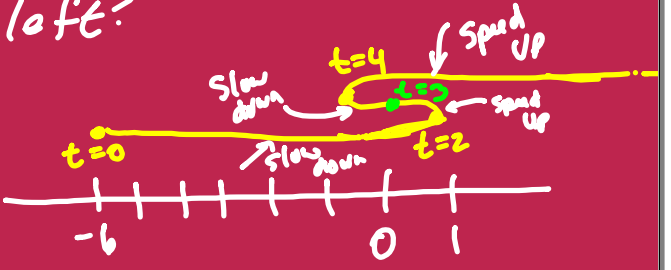
When is moving to right? left?

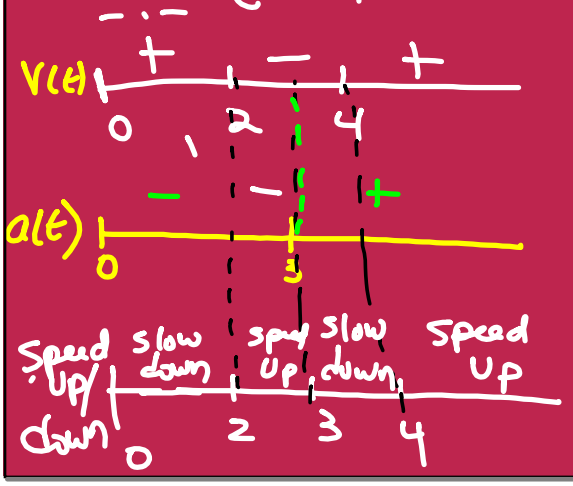
$v(t) = t^2 - 6t + 8 = 0$

$(t-2)(t-4) = 0$

$t = 2, 4$

t	s
0	-6
2	$2/3$
4	$-2/3$





speed up = v + a go in same direction
(2,3) (4,∞)

Slow down = v + a go in opposite direction
(0,2) (3,4)

$a(t) = 2t - 6 = 0$

$t = 3$

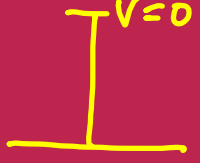
$h(t) = \frac{1}{2}at^2 + V_0t + S_0$

$a = -32$

$s(t) = h(t) = -16t^2 + 64t + 2$

$v(t) = -32t + 64$

$a(t) = -32$



present = 64 ft/s

$a = -32\ ft/s^2$

$a = -9.8\ m/s^2$

How high will the present go?

$0 = -32t + 64$

$32t = 64$

$t = 2$ sec

$h(2) = -16(2)^2 + 64(2) + 2$

$-64 + 128 + 2 = 66$ ft.

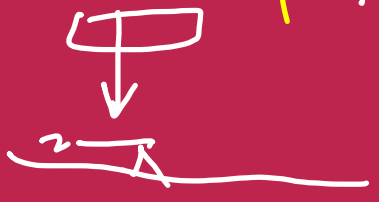
At what velocity should the elf throw the present?

$128 = -16t^2 + vt + 2$

$128 = -64 + 2v + 2$

$190 = 2v$

$95 = v$



Santa throws snowball downward at 96 ft/sec. How long does the elf have to move out of way?

$h(t) = -16t^2 - 96t + 128$

$2 = -16t^2 - 96t + 128$

$0 = -16t^2 - 96t + 126$

$t = 1.1$ ~~$t = -7.1$~~

$-131.2 \frac{ft}{sec} \cdot \frac{1\ mi}{5280\ ft} \cdot \frac{3600\ sec}{1\ hr}$

$= -89$ mph

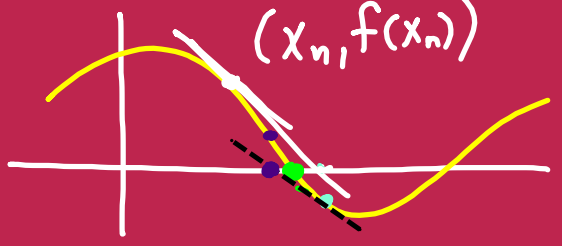
Velocity of snowball

$v(t) = -32t - 96$

$v(1.1) = -131.2\ ft/s$

December 15, 2021

Newton's Method



$f(x) = x^3 + x - 1 \quad [-4, 1]$

$$x - \frac{x^3 + x - 1}{3x^2 + 1} \Bigg| \; x = 0$$

$$y - f(x_n) = f'(x_n)(x - x_n)$$

$$0 - f(x_n) = f'(x_n)(x - x_n)$$

$$\frac{-f(x_n)}{f'(x_n)} = x - x_n$$

$$\boxed{x_n - \frac{f(x_n)}{f'(x_n)} = x}$$