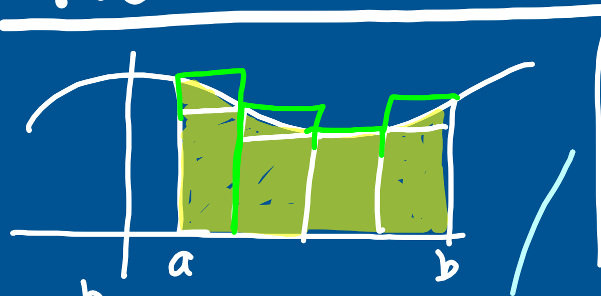
RIEMANN Sums

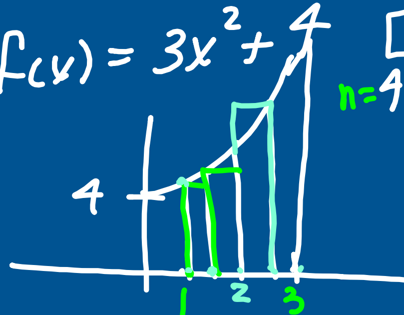


$$\lim_{\Delta x \to 0} \sum_{x=a}^{b} f(x)\,\Delta x$$

↑ width

$f(x) = 3x^2 + 4$ a b [1, 3]

n = 4 sub-intervals



$$\text{width} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} = \frac{b-a}{n}$$

$f(x) = 3x^2 + 4$

$$\text{Left} = \tfrac{1}{2}\cdot f(1) + \tfrac{1}{2}f(1.5) + \tfrac{1}{2}f(2) + \tfrac{1}{2}f(2.5)$$

$$= \tfrac{1}{2}\left[f(1) + f(1.5) + f(2) + f(2.5)\right]$$

$$= \tfrac{1}{2}\left[7 + 10.75 + 16 + 22.75\right]$$

$$= \tfrac{1}{2}\left[56.5\right]$$

$$\boxed{\text{Left} = 28.25}$$

Right =

$$= \tfrac{1}{2}\left[f(1.5) + f(2) + f(2.5) + f(3)\right]$$

$$= \tfrac{1}{2}\left[10.75 + 16 + 22.75 + 31\right]$$

$$= \tfrac{1}{2}\left[80.5\right]$$

$$= 40.25$$

Definite Integrals

results in a numerical value

Indefinite Integrals

function + C

$$\int_{-2}^{5}(4x+3)\,dx$$

$$= \frac{4x^2}{2} + 3x + C \Big|_{-2}^{5}$$

$$= 2x^2 + 3x + C \Big|_{-2}^{5}$$

$$= 50 + 15 + \cancel{C} + (-8 + {}^{+}6 + \cancel{-C})$$

$$= 63$$

$$\int_{5}^{-2}(4x+3)\,dx = -\int_{-2}^{5}(4x+3)\,dx$$

$$\int_{1}^{6} x\sqrt{x+3}\,dx$$

$u = x + 3 \Rightarrow u - 3 = x$

$du = 1\,dx$

$$\int_{4}^{9}(u^1 - 3)\,u^{1/2}\,du$$

$$\int_{4}^{9}(u^{3/2} - 3u^{1/2})\,du$$

$$\frac{2}{5}u^{5/2} - \frac{2}{\cancel{3}}\cancel{3}u^{3/2}\Big|_{4}^{9}$$

$$\frac{2}{5}u^{5/2} - 2u^{3/2}\Big|_{4}^{9} \qquad \sqrt{9}^{3}$$

$$\frac{2}{5}\cdot 243 - 2\cdot 27 - \left[\frac{2}{5}\cdot 32 - 2\cdot 8\right]$$

$$\frac{486}{5} - 54 - \frac{64}{5} + 16$$

$$= \frac{422}{5} - \frac{38}{5}$$

$$= \frac{422}{5} - \frac{190}{5}$$

$$= \frac{232}{5}$$

When using u-substitution - Change the limits of integration & sub into the u's!

Fundamental Theorem of Calculus

Part 1

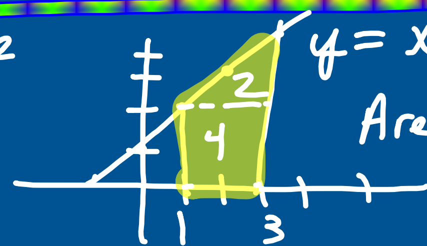
$$\int_1^3 (x+1) = \frac{x^2}{2} + x \Big|_1^3$$

$$= \frac{9}{2} + 3 - \left(\frac{1}{2} + 1\right)$$

$$= 4 + 2$$

$$= 6$$

$\frac{1}{2} \cdot 2 \cdot 2$



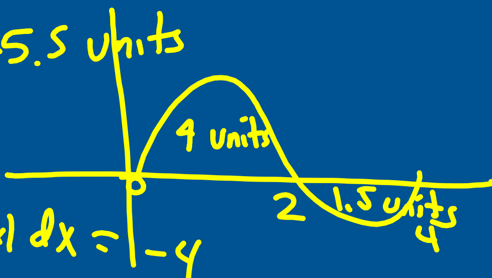
$y = x + 1$

Area = 6 Units²

Integration represents the area between a curve + an axis.

$$\int_0^2 f(x)\,dx = 4$$

$$\int_0^4 f(x) = 5.5 \text{ units}$$



$$\int_2^0 f(x)\,dx = -4$$

Part 2

$$\frac{d}{dx}\int_1^x (4t^2+t)\,dt = \boxed{4x^2+x}$$

$$\frac{d}{dx}\left[\frac{4t^3}{3}+\frac{t^2}{2}\right|_1^x = \frac{d}{dx}\left[\frac{4x^3}{3}+\frac{x^2}{2}-\left[\frac{1}{3}+\frac{1}{2}\right]\right]$$

$$\frac{d}{dx}\int_6^x \frac{\sin^8(3t^2-1)}{\ln 8t^4}\,dt = \frac{\sin^8(3x^2-1)}{\ln 8x^4}$$

$$\frac{d}{dx}\int_2^{x^4} \frac{t^4}{\sqrt{t^3+2}}\,dt = \frac{(x^4)^4}{\sqrt{(x^4)^3+2}}\cdot 4x^3 = \frac{x^{16}}{\sqrt{x^{12}+2}}\cdot 4x^3$$

$$= \frac{4x^{19}}{\sqrt{x^{12}+2}}$$

$$\frac{d}{dx}\int_{x^4}^{3x^7} \frac{2t}{t+1}\,dt$$

$$\frac{6x^7}{3x^7+1}\cdot 21x^6 - \frac{2x^4}{x^4+1}\cdot 4x^3$$

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