ANTIDIFFERENTIATION

$y = f(x)$

$\frac{dy}{dx} = f'(x)$

$\int dy = \int f'(x)\,dx$

$y = \int f'(x)\,dx$

$\int (4x + 9x^2)\,dx$

$\frac{4x^2}{2} \quad \frac{9x^3}{3}$

$= 2x^2 + 3x^3 + C$

Power Rule

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

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$$\int \left(\frac{2}{x^3} + 4\sqrt[3]{x} - x^{3/5} + 7 \right) dx$$

$$\int \left(2x^{-3} + 4x^{1/3} - x^{3/5} + 7 \right) dx$$

$$= \frac{2x^{-2}}{-2} + \frac{3}{\cancel{4}} \cdot \cancel{4} x^{4/3} - \frac{5}{8} x^{8/5} + 7x + C$$

$$= \frac{-1}{x^2} + 3x^{4/3} - \frac{5}{8} x^{8/5} + 7x + C$$

$$\int (7x-4)^2 \, dx$$

$$\int \left(49x^2 - 56x + 16 \right) dx$$

$$= \frac{49x^3}{3} - \frac{56x^2}{2} + 16x + C$$

$$= \frac{49}{3} x^3 - 28x^2 + 16x + C$$

$$\int \frac{4x^2-2x+1}{\sqrt{x}}\,dx$$

$$\int \left(4x^2-2x^1+1\right)\cdot x^{-1/2}\,dx$$

$$\int \left(4x^{3/2}-2x^{1/2}+x^{-1/2}\right)dx$$

$$\frac{2}{5}\cdot 4x^{5/2}-\frac{2}{3}\cdot 2x^{3/2}+2x^{1/2}+C$$

$$\boxed{\frac{8}{5}x^{5/2}-\frac{4}{3}x^{3/2}+2x^{1/2}+C}$$

Initial Value Problems ← (Find C.)

Find y.

$$\int \frac{dy}{dx} = \int (3x^2+2x)\,dx \qquad y(2)=7$$

$$y = \cancel{3}\frac{x^3}{\cancel{3}} + \cancel{2}\frac{x^2}{\cancel{2}} + C$$

$$7 = 2^3 + 2^2 + C$$

$$7 = 8+4+C$$

$$-12$$

$$-5 = C$$

$$\boxed{y = x^3+x^2-5}$$

U-Substitution (Reverse chain rule)

$$\int 6x(x^2+5)^8\,dx$$

$u = x^2+5$

$\frac{du}{dx} = 2x\,dx$

$\frac{du}{2x} = dx$

$$\int \overset{3}{\cancel{6x}}\, u^8 \cdot \frac{du}{\cancel{2x}}$$

$$= \frac{3u^9}{9} + C$$

$$= \boxed{\frac{1}{3}(x^2+5)^9 + C}$$

$3(x^2+5)^8 \cdot 2x$

$$\int \frac{3x^{10}}{(4-6x^{11})^7}\,dx$$

$u = 4-6x^{11}$

$du = -66x^{10}\,dx$

$\frac{du}{-66x^{10}} = dx$

$$\int \frac{\cancel{3}\cancel{x^{10}}}{u^7} \cdot \frac{du}{-\underset{-22}{\cancel{66}}\cancel{x^{10}}}$$

$$\frac{1}{-22}\int \frac{1}{u^7}\,du$$

$$-\frac{1}{22}\int u^{-7}\,du$$

$$-\frac{1}{22} \cdot \frac{u^{-6}}{-6} + C$$

$$\boxed{\frac{1}{132(4-6x^{11})^6} + C}$$

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$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int (4\cos x - 3\sec^2 x + 4\csc x \cot x)\,dx$$

$$4\sin x - 3\tan x - 4\csc x + C$$

$$\int \csc x\,(\csc x - \cot x)\,dx$$

$$\int (\csc^2 x - \csc x \cot x)\,dx$$

$$-\cot x + \csc x + C$$

$$\int \left(\frac{1}{\sin^2 x} - \cot x \sin x\right) dx$$

$$\int \left(\csc^2 x - \frac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x}\right) dx$$

$$\int (\csc^2 x - \cos x)\,dx$$

$$(6x-5)e^{3x^2-5x}$$

$$-\cot x - \sin x + C$$

$$\int \left(\frac{6}{x} + 5e^x\right) dx$$

$$\cancel{\int 6x^{-1} = \frac{6x^0}{0}}$$

$$6 \cdot \frac{1}{x}$$

$$= \boxed{6\ln|x| + 5e^x + C}$$