

Integration 3 - exponential, logarithmic & inverse trig functions

$$\frac{d}{dx} e^x = e^x \quad \int e^x dx$$

$$\frac{d}{dx} a^x = \ln a \cdot a^x \quad \int a^x dx = \frac{1}{\ln a} a^x + C$$

$$\frac{d}{dx} \ln x = \frac{1}{x} \quad \int \frac{1}{x} dx = \ln|x| + C$$

$$\int x \cdot e^{5x^2} dx$$

$$\int x \cdot e^u \cdot \frac{du}{10x}$$

to $\int e^u du$

$$\frac{1}{10} e^u + C$$

$$\boxed{\frac{1}{10} e^{5x^2} + C}$$

$$u = 5x^2$$

$$du = 10x dx$$

$$\frac{du}{10x} = dx$$

$$\int \frac{e^{\tan y}}{\cos^2 y} dy \quad u = \tan y$$

$$du = \sec^2 y dy$$

$$\frac{du}{\sec^2 y} dy$$

$$\int \frac{e^u \cdot \sec^2 y}{\sec^2 y} \cdot \frac{du}{\sec^2 y}$$

$$= e^u + C$$

$$= e^{\tan y} + C$$

$$\int 3x^5 \cdot 7^{x^6} dx$$

$$u = x^6$$

$$du = 6x^5 dx$$

$$\frac{du}{6x^5} = dx$$

$$\int a^x dx = \frac{1}{\ln a} \cdot a^x$$

$$\int \cancel{3x^5} \cdot 7^u \cdot \frac{du}{\cancel{6x^5}}$$

$$\frac{1}{2} \int 7^u du$$

$$= \frac{1}{2} \cdot \frac{1}{\ln 7} \cdot 7^u + C$$

$$= \boxed{\frac{1}{2 \ln 7} \cdot 7^{x^6} + C}$$

$$= \frac{7^{x^6}}{2 \ln 7} + C \quad \text{or} \quad \frac{7^{x^6}}{\ln 49} + C$$

$$\int \frac{x^2}{1-x^3} dx \quad \begin{array}{l} u = 1-x^3 \\ du = -3x^2 dx \end{array}$$

$$\int \frac{\cancel{x^2}}{u} \frac{du}{-3\cancel{x^2}}$$

$$\begin{aligned} & -\frac{1}{3} \int \frac{1}{u} du \\ & = -\frac{1}{3} \ln|u| + C \\ & = -\frac{1}{3} \ln|1-x^3| + C \end{aligned}$$

$$\int \frac{\csc^2 x}{\cot x} dx \quad \begin{array}{l} u = \cot x \\ du = -\csc^2 x dx \end{array}$$

$$\int \frac{\csc^2 x}{u} \frac{du}{-\csc^2 x}$$

$$\begin{aligned} & - \int \frac{1}{u} du \\ & = -\ln|u| + C \\ & = -\ln|\cot x| + C \end{aligned}$$

$$\int \frac{(\ln z)^5}{z} dz \quad \begin{array}{l} u = \ln z \\ du = \frac{1}{z} dz \\ z du = dz \end{array}$$

$$\int \frac{u^5}{\cancel{z}} \cdot \cancel{z} du$$

$$= \frac{u^6}{6} + C$$

$$= \frac{(\ln z)^6}{6} + C$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \tan^{-1} x = \frac{1}{x^2+1} \quad \frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\int \frac{6x^2}{\sqrt{1-\frac{25x^6}{(5x^3)^2}}} dx$$

$$u = 5x^3$$

$$du = 15x^2 dx$$

$$\int \frac{6x^2}{\sqrt{1-u^2}} \frac{du}{15x^2}$$

$$\frac{du}{15x^2} = dx$$

$$\frac{2}{5} \int \frac{1}{\sqrt{1-u^2}} du$$

$$\int \frac{3x}{4+9x^2} dx \quad \text{1) Make the "1"$$

$$\frac{2}{5} \sin^{-1} u + C$$

$$\frac{3}{4} \int \frac{x}{1+\frac{9}{4}x^2} dx$$

$$u = \frac{3}{2}x^2$$

$$du = 3x dx$$

$$\frac{du}{3x} = dx$$

$$\frac{2}{5} \sin^{-1}(5x^3) + C$$

$$\frac{3}{4} \int \frac{x}{1+u^2} \cdot \frac{du}{3x}$$

$$\frac{1}{4} \int \frac{1}{1+u^2} du$$

$$= \frac{1}{4} \tan^{-1} u + C$$

$$= \frac{1}{4} \tan^{-1} \left(\frac{3}{2}x^2 \right) + C$$

$$\int \frac{4 \cos x}{\sin x \sqrt{\sin^2 x - 36}} dx$$

$$\frac{4}{6} \int \frac{\cos x}{\sin x \sqrt{\frac{\sin^2 x}{36} - 1}} dx$$

$$\frac{2}{3} \int \frac{\cancel{\cos x}}{\sin x \sqrt{u^2 - 1}} \frac{6}{\cancel{\cos x}} du$$

$$4 \int \frac{1}{\sin x \sqrt{u^2 - 1}} du$$

$$\frac{4}{6} \int \frac{1}{\cancel{2}u \sqrt{u^2 - 1}} du$$

$$\frac{2}{3} \sec^{-1} u + C$$

$$\frac{2}{3} \sec^{-1} \left(\frac{\sin x}{6} \right) + C$$

$$u = \frac{\sin x}{6} = \frac{1}{6} \sin x$$

$$du = \frac{1}{6} \cos x dx$$

$$\frac{6}{\cos x} du = dx$$

$$u = \frac{\sin x}{6}$$

$$6u = \sin x$$