FOCUS ON CONCEPTS

7. Explain the connection between the chain rule for differentiation and the method of u-substitution for integration.
8. Explain how the substitution $u = ax + b$ helps to perform an integration in which the integrand is $f(ax + b)$, where $f(x)$ is an easy to integrate function.

9–50 Evaluate the integrals using appropriate substitutions.

9. $\int (4x - 3)^9\,dx$
10. $\int x^3\sqrt{5 + x^4}\,dx$
11. $\int \sin 7x\,dx$
12. $\int \cos\frac{x}{3}\,dx$
13. $\int \sec 4x \tan 4x\,dx$
14. $\int \sec^2 5x\,dx$
15. $\int e^{2x}\,dx$
16. $\int \frac{dx}{2x}$
17. $\int \frac{dx}{\sqrt{1 - 4x^2}}$
18. $\int \frac{dx}{1 + 16x^2}$
19. $\int t\sqrt{7t^2 + 12}\,dt$
20. $\int \frac{x}{\sqrt{4 - 5x^2}}\,dx$
21. $\int \frac{6}{(1 - 2x)^3}\,dx$
22. $\int \frac{x^2 + 1}{\sqrt{x^3 + 3x}}\,dx$
23. $\int \frac{x^3}{(5x^4 + 2)^3}\,dx$
24. $\int \frac{\sin(1/x)}{3x^2}\,dx$
25. $\int e^{\sin x}\cos x\,dx$
26. $\int x^3 e^{x^4}\,dx$
27. $\int x^2 e^{-2x^3}\,dx$
28. $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$
29. $\int \frac{e^x}{1 + e^{2x}}\,dx$
30. $\int \frac{t}{t^4 + 1}\,dt$
31. $\int \frac{\sin(5/x)}{x^2}\,dx$
32. $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx$
33. $\int \cos^4 3t \sin 3t\,dt$
34. $\int \cos 2t \sin^5 2t\,dt$
35. $\int x \sec^2(x^2)\,dx$
36. $\int \frac{\cos 4\theta}{(1 + 2\sin 4\theta)^4}\,d\theta$
37. $\int \cos 4\theta\sqrt{2 - \sin 4\theta}\,d\theta$
38. $\int \tan^3 5x \sec^2 5x\,dx$
39. $\int \frac{\sec^2 x\,dx}{\sqrt{1 - \tan^2 x}}$
40. $\int \frac{\sin\theta}{\cos^2\theta + 1}\,d\theta$
41. $\int \sec^3 2x \tan 2x\,dx$
42. $\int [\sin(\sin\theta)]\cos\theta\,d\theta$
43. $\int \frac{dx}{e^x}$
44. $\int \sqrt{e^x}\,dx$
45. $\int \frac{dx}{\sqrt{x}\,e^{(2\sqrt{x})}}$
46. $\int \frac{e^{\sqrt{2y+1}}}{\sqrt{2y + 1}}\,dy$
47. $\int \frac{y}{\sqrt{2y + 1}}\,dx$
48. $\int x\sqrt{4 - x}\,dx$
49. $\int \sin^3 2\theta\,d\theta$
50. $\int \sec^4 3\theta\,d\theta$ [*Hint:* Apply a trigonometric identity.]

51–54 Evaluate each integral by first modifying the form of the integrand and then making an appropriate substitution, if needed.

51. $\int \frac{t + 1}{t}\,dt$
52. $\int e^{2\ln x}\,dx$
53. $\int [\ln(e^x) + \ln(e^{-x})]\,dx$
54. $\int \cot x\,dx$

55–56 Evaluate the integrals with the aid of Formulas (5), (6) and (7).

55. (a) $\int \frac{dx}{\sqrt{9 - x^2}}$ (b) $\int \frac{dx}{5 + x^2}$ (c) $\int \frac{dx}{x\sqrt{x^2 - \pi}}$
56. (a) $\int \frac{e^x}{4 + e^{2x}}\,dx$ (b) $\int \frac{dx}{\sqrt{9 - 4x^2}}$ (c) $\int \frac{dy}{y\sqrt{5y^2 - 3}}$

57–59 Evaluate the integrals assuming that n is a positive integer and $b \neq 0$.

57. $\int (a + bx)^n\,dx$
58. $\int \sqrt[n]{a + bx}\,dx$
59. $\int \sin^n(a + bx)\cos(a + bx)\,dx$

C 60. Use a CAS to check the answers you obtained in Exercises 57–59. If the answer produced by the CAS does not match yours, show that the two answers are equivalent. [*Suggestion: Mathematica* users may find it helpful to apply the Simplify command to the answer.]

FOCUS ON CONCEPTS

61. (a) Evaluate the integral $\int \sin x \cos x\,dx$ by two methods: first by letting $u = \sin x$, and then by letting $u = \cos x$.
    (b) Explain why the two apparently different answers obtained in part (a) are really equivalent.
62. (a) Evaluate the integral $\int (5x - 1)^2\,dx$ by two methods: first square and integrate, then let $u = 5x - 1$.
    (b) Explain why the two apparently different answers obtained in part (a) are really equivalent.

▶ Exercise Set 6.3 (Page 371)

1. (a) $\frac{(x^2+1)^{24}}{24}+C$ (b) $-\frac{\cos^4 x}{4}+C$
   (c) $-2\cos\sqrt{x}+C$ (d) $\frac{3}{4}\sqrt{4x^2+5}+C$
3. (a) $-\frac{1}{2}\cot^2 x+C$ (b) $\frac{1}{10}(1+\sin t)^{10}+C$
   (c) $\frac{1}{2}\sin 2x+C$ (d) $\frac{1}{2}\tan(x^2)+C$
5. (a) $\ln|\ln x|+C$ (b) $-\frac{1}{5}e^{-5x}+C$
   (c) $-\frac{1}{3}\ln(1+\cos 3\theta)+C$ (d) $\ln(1+e^x)+C$
9. $\frac{1}{40}(4x-3)^{10}+C$ 11. $-\frac{1}{7}\cos 7x+C$ 13. $\frac{1}{4}\sec 4x+C$
15. $\frac{1}{2}e^{2x}+C$ 17. $\frac{1}{2}\sin^{-1}(2x)+C$ 19. $\frac{1}{21}(7t^2+12)^{3/2}+C$
21. $\frac{3}{2(1-2x)^2}+C$ 23. $-\frac{1}{40(5x^4+2)^2}+C$ 25. $e^{\sin x}+C$
27. $-\frac{1}{6}e^{-2x^3}+C$ 29. $\tan^{-1}e^x+C$ 31. $\frac{1}{5}\cos(5/x)+C$
33. $-\frac{1}{15}\cos^5 3t+C$ 35. $\frac{1}{2}\tan(x^2)+C$ 37. $-\frac{1}{6}(2-\sin 4\theta)^{3/2}+C$
39. $\sin^{-1}(\tan x)+C$ 41. $\frac{1}{6}\sec^3 2x+C$ 43. $-e^{-x}+C$
45. $-e^{-2\sqrt{x}}+C$ 47. $\frac{1}{6}(2y+1)^{3/2}-\frac{1}{2}(2y+1)^{1/2}+C$
49. $-\frac{1}{2}\cos 2\theta+\frac{1}{6}\cos^3 2\theta+C$ 51. $t+\ln|t|+C$
53. $\int[\ln(e^x)+\ln(e^{-x})]dx=C$
55. (a) $\sin^{-1}\left(\frac{1}{3}x\right)+C$ (b) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+C$
   (c) $\frac{1}{\sqrt{\pi}}\sec^{-1}\left(\frac{x}{\sqrt{\pi}}\right)+C$
57. $\frac{1}{b}\frac{(a+bx)^{n+1}}{n+1}+C$ 59. $\frac{1}{b(n+1)}\sin^{n+1}(a+bx)+C$
61. (a) $\frac{1}{2}\sin^2 x+C_1;\ -\frac{1}{2}\cos^2 x+C_2$ (b) They differ by a constant.
63. $\frac{2}{15}(5x+1)^{3/2}-\frac{158}{15}$ 65. $y=-\frac{1}{2}e^{2t}+\frac{13}{2}$
67. (a) $\sqrt{x^2+1}+C$ 69. $f(x)=\frac{2}{9}(3x+1)^{3/2}+\frac{7}{9}$

24. $\frac{1}{3}\cos\left(\frac{1}{x}\right)+C$

36. $\frac{-1}{24(1+2\sin 4\theta)^3}+C$

38. $\frac{1}{20}\tan^4(5x)+C$

42. $-\cos(\sin\theta)+C$

CALCULUS HANDOUT
Integration of Exponential, Logarithmic & Inverse Trig Functions

1. $\int e^{\sin x}\cos x\,dx$
2. $\int 6^{x}\,dx$
3. $\int \frac{1}{1+16x^{2}}dx$
4. $\int x^{2}e^{-2x^{3}}\,dx$
5. $\int \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\,dx$
6. $\int \frac{4}{x\ln x\sqrt{(\ln x)^{2}-1}}dx$
7. $\int x\cdot 2^{5x^{2}}\,dx$
8. $\int \frac{1}{e^{x}}dx$
9. $\int \frac{3x^{2}}{\sqrt{4-25x^{6}}}dx$
10. $\int \frac{x^{2}}{x^{3}-4}dx$
11. $\int \pi^{\sin x}\cos x\,dx$
12. $\int \frac{1}{x\sqrt{9x^{2}-1}}dx$
13. $\int \frac{1}{x\ln x}dx$
14. $\int \frac{e^{x}}{4+e^{2x}}dx$
15. $\int \frac{\sin 3\theta}{1+\cos 3\theta}d\theta$
16. $\int \frac{1}{\sqrt{9-4x^{2}}}dx$

Evaluate each integral by first modifying the form of the integrand and then making an appropriate substitution, if needed.

17. $\int \frac{t+1}{t}dt$
18. $\int e^{2\ln x}\,dx$
19. $\int\left[\ln\left(e^{x}\right)+\ln\left(e^{-x}\right)\right]\,dx$
20. $\int \cot x\;dx$

ANSWERS

1. $e^{\sin x}+C$

2. $\frac{1}{\ln 6}\cdot 6^{x}+C$

3. $\frac{1}{4}\tan^{-1}4x+C$

4. $-\frac{1}{6}e^{-2x^{3}}+C$

5. $\ln\left|e^{x}-e^{-x}\right|+C$

6. $4\sec^{-1}\left(\ln x\right)+C$

7. $\frac{1}{10\ln 2}\cdot 2^{5x^{2}}+C$

8. $-e^{-x}+C$

9. $\frac{1}{5}\sin^{-1}\left(\frac{5x^{3}}{2}\right)+C$

10. $\frac{1}{3}\ln\left|x^{3}-4\right|+C$

11. $\frac{1}{\ln\pi}\cdot\pi^{\sin x}+C$

12. $\sec^{-1}\left(3x\right)+C$

13. $\ln\left|\ln x\right|+C$

14. $\frac{1}{2}\tan^{-1}\left(\frac{e^{x}}{2}\right)+C$

15. $-\frac{1}{3}\ln\left|1+\cos 3\theta\right|+C$

16. $\frac{1}{2}\sin^{-1}\left(\frac{2}{3}x\right)+C$

17. $t+\ln t+C$

18. $\frac{x^{3}}{3}+C$

19. C

20. $\ln\left|\sin x\right|+C$