Newton and Leibniz: the Calculus Controversy

The History of Calculus

The history of calculus does not begin with Newton and Leibniz's findings. Their calculus was the culmination of centuries of work by other mathematicians rather than an instant epiphany that came individually to them. Below is a summary of some of the more important developments in calculus up until the early seventeenth century.

1600 B.C

The Rhind Papyrus shows that the ancient Egyptians knew that the volume of a rectangular pyramid volume is equivalent to 1/3 of the volume of a rectangular prism having the same base and height, but not how they reached this conclusion. The Babylonians were able to devise a formula for the value of a square root of any rational number to as many decimal places as was desired. The Babylonians did not realize that this was an infinite process.

425-200 B.C.

The ancient Greeks discovered that the diagonal of many geometric figures would never be a whole number and that determining this value involved an infinite process. The Greek Eudoxus developed the "method of exhaustion" and used his method to prove the Egyptian discovery about the area of a pyramid as well as formulas for the volume of many other geometric figures. The Greek Archimedes developed his own method of determining the area of a segment of a parabola.

320 A.D.

The Greek mathematician Pappus determined that the volume generated by the rotation of a plane figure about an axis not cutting the figure is equal to the product of the area of the plane figure and the distance that the center of gravity of the plane figure covers in the revolution." Unfortunately, Pappus did not provide a proof of his statement.

300-1300 A.D.

Europe and mathematics experienced one thousand years of decline during the medieval times.

1300 A.D.

Nicole Oresme constructed one of the first graphs of a function. He went so far as to say that the area under the graph would represent the total distance covered and was even able to integrate functions that took the form of triangles, rectangles, or trapezoids.

1630 A.D.

Bonaventura Cavalieri published his discoveries concerning analytic geometry and indivisibles and, later, an equivalent to what we now call the power rule for integration.

1620-1640 A.D.

Pierre de Fermat created a more logically adequate process for calculating integrals. He was the first mathematician to be credited with discovering the process and power rule for differentiation. Fermat was one of many to notice the inverse relationship between integrals and derivatives but not the importance of this relationship.

By the early seventeenth century, everything was in place for Newton and Leibniz to take all these ideas of limited scope and compile them into methods of universal applicability.

Isaac Newton

Sir Isaac Newton was born on December 25, 1642 in Woolsthorpe, England. He attended the King's School at Gratham and went on to pursue a higher education at Cambridge University. He graduated in 1665 without honors or distinction. He obtained his masters degree in 1668. Newton made discoveries in mathematics, optics, and physics before his death on March 20, 1727.

Calculus and Notation

Using infinite series and the already established power rule for integrals, Newton was able to calculate the areas under curves that others previously could not. He was also able to calculate the tangent lines to these curves. He called his calculus the "method of fluxions" and he thought of everything in terms of motion. He considered the dependent variable x to be the "fluent" and its velocity to be the "fluxion." He designated the fluxion, nowadays known as the derivative with respect time t, with the notation $\dot{x}$. He called the differential of x the "moment" of x and designated it with the notation $\dot{x}o$, which represented the change in the velocity of x in an infinitely small time period. He designated the fluent, or antiderivative of x, first by $\square x$ or $\boxed{x}$ and later by $\overset{|}{x}$.

To calculate the derivative of the function $y = \frac{5x^2+2}{7}$, Newton would first solve this equation for 0, leaving him with $7y - 5x^2 - 2 = 0$. Next, Newton would plug in $x + \dot{x}o$ for x and $y + \dot{y}o$ for y, leaving him with $7(y + \dot{y}o) - 5(x + \dot{x}o)^2 - 2$. Newton would subtract the original equation from the change equation as follows:

$$
\begin{aligned}
7(y + \dot{y}o) - 5(x + \dot{x}o)^2 - 2 - (7y - 5x^2 - 2) &= 0 \\
7y + 7\dot{y}o - 5(x^2 + 2x\dot{x}o + \dot{x}^2o^2) - 2 - 7y + 5x^2 + 2 &= 0 \\
7y + 7\dot{y}o - 5x^2 - 10x\dot{x}o - 5\dot{x}^2o^2 - 2 - 7y + 5x^2 + 2 &= 0 \\
7\dot{y}o - 10x\dot{x}o - 5\dot{x}^2o^2 &= 0
\end{aligned}
$$

Next, Newton would divide through by o, the "moment" of x, leaving:

$$7\dot{y} - 10x\dot{x} - 5\dot{x}^2 o = 0$$

Newton claimed that terms with o were nothing compared to the others and could be cast out:

$$7\dot{y} - 10x\dot{x} = 0$$

Newton's method found the differential equation that would satisfy a given equation. However, this differential equation can easily be solved for the derivative, $\frac{\dot{y}}{\dot{x}}$, as follows:

$$
\begin{aligned}
7\dot{y} - 10x\dot{x} &= 0 \\
7\dot{y} &= 10x\dot{x} \\
\dot{y} &= \frac{10x\dot{x}}{7} \\
\frac{\dot{y}}{\dot{x}} &= \frac{10x}{7}
\end{aligned}
$$

Gottfried Wilhelm von Leibniz

Gottfried Wilhelm von Leibniz was born on July 1, 1646 in Leipzig, Germany. He attended the Nicolai school, but he was largely self-taught. He studied law at the University of Leipzig but the school refused to grant him a doctorate at the young age of twenty-one, so he obtained it from the University at Altdorf instead. Leibniz made discoveries in mathematics and physics before his death on November 14, 1716.

Calculus and Notation

While Newton thought of calculus in terms of motion, Leibniz viewed it in terms of sums and differences. Specifically, Leibniz used ordinates and sequences of the differences of these ordinates to calculate the area under curves. Following these discoveries, Leibniz introduced the notation $\int x\,dx$, where $\int$ was an enlongated representation of the first letter of the Latin word *summa*, meaning summation, and d was the first letter of the Latin word *differentia*, meaning differential (infinitesimal distance). Leibniz also used a differential triangle to discover the slope of a tangent line to a curve. He was thus able to derive the power, product, quotient, and chain rules.

Leibniz calculated the derivative of the function $y = \frac{5x^2+2}{7}$ differently than Newton but in a manner familiar to the modern-day calculus student. Having discovered the power rule $d(x^n) = nx^{n-1}dx$ and that the derivative of a constant is 0, Leibniz was able to take the derivative of the function as follows:

$$
\begin{aligned}
y &= \frac{1}{7}(5x^2 + 2) \\
dy &= d(\frac{1}{7}(5x^2 + 2)) \\
dy &= \frac{1}{7}(d(5x^2) + d(2)) \\
dy &= \frac{1}{7}(5 * 2x + 0)dx \\
dy &= \frac{10x}{7}dx \\
\frac{dy}{dx} &= \frac{10x}{7}
\end{aligned}
$$

The Controversy

The calculus controversy emerged largely due to the timing of these men's publications. While Newton had made his discoveries in 1664-1666, his findings were not published until 1693. Leibniz, on the other hand, made his discoveries after Newton, in the timeframe of 1672-1676, but published them in 1684 and 1686, before Newton. The differences between the discovery dates and publication dates led the mathematical community to question whether Leibniz had truly discovered the method independently of Newton, or if he had merely stolen Newtons ideas and coupled them with his own unique notation.

National pride played a great role in the exacerbation of the dispute. Those involved realized that credit for the discovery of a whole new branch of mathematics was at stake, and each side wanted their country to get this credit. In 1711, the controversy was taken to court. A commission was appointed by the Royal Society to look into the charges. Since Newton was the president of the Royal Society, it is not all that surprising that Leibniz was found guilty of plagiarism.

Eventually, the mathematical community came to realize that Newton and Leibniz had made their discoveries independently, but not until years after Leibniz's death. During this time, continental Europe continued to use Leibniz's easier notation and methods while England remained loyal to the more complicated methods and notation of their own Newton. For this reason, England was far behind the rest of the continent in mathematics for the entire eighteenth century.

Conclusion

The calculus controversy may seem frivolous to the modern reader, but it is necessary to recognize its importance at the time. This controversy was far more than just a quibble between two mathematicians who wanted credit for the discovery of calculus; indeed, it was a matter of national pride and historical significance. As a piece of history, the controversy serves as a lesson to the modern world that it is perhaps better for great minds to work together instead of trying to undermine each other. This can aid in the avoidance of stagnation in mathematical and scientific thought and advances.