DOUBLE + HALF ANGLE IDENTITIES

$\sin(\cancel{A}+\cancel{B}) = \sin\cancel{A}\cos\cancel{B} + \cos\cancel{A}\sin\cancel{B}$

$\sin 2x = 2\sin x\cos x$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\cos A = 1 - 2\sin^2 \frac{A}{2}$

$\dfrac{\cancel{2}\sqrt{\sin^2 \frac{A}{2}}}{\cancel{2}} = \sqrt{\dfrac{1-\cos A}{2}}$

$\sin \frac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}$

$\cos \frac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$

$\tan \frac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}} \dfrac{(1+\cos A)}{(1+\cos A)}$ $1-\cos^2 A$

$* = \dfrac{1-\cos A}{\sin A}$

$= \dfrac{\sin A}{1+\cos A}$

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True/False

T $\cos 50^\circ = 1 - 2\sin^2 25^\circ$

$= \cos(2 \cdot 25^\circ)$

$= \cos 50^\circ$

F $\sin 92^\circ = 2\sin 84^\circ \cdot \cos 84^\circ$

$= \sin(2 \cdot 84^\circ)$

QII

True $\cos 130^\circ = -\sqrt{\dfrac{1-\cos 260^\circ}{2}}$

$= \sin \dfrac{A}{2} = \sin\left(\dfrac{260^\circ}{2}\right) = \sin 130^\circ$

determined by the quadrant where $\frac{A}{2}$ is located.

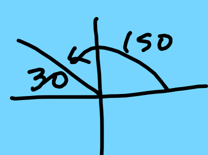
F $\sin 194^\circ = -\sqrt{\dfrac{1-\cos 388^\circ}{2}}$

QIII

Evaluate

$$\frac{2\tan 75^\circ}{1-\tan^2 75^\circ} = \tan(2\cdot 75^\circ)$$

$$= \tan 150^\circ$$



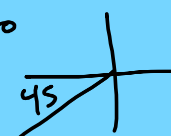
$$= \boxed{-\frac{\sqrt{3}}{3}}$$

Evaluate

$$\frac{1-\cos 450^\circ}{\sin 450^\circ}$$

$$= \tan\left(\frac{A}{2}\right) = \tan\left(\frac{450^\circ}{2}\right)$$

$$= \tan 225^\circ$$



$$= +1$$

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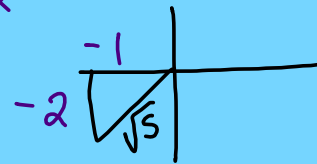
Find $\sin 2x$ given $\tan x = \frac{2}{1} \frac{y}{x}$ & x in Q III.



$$\sin 2x = 2\sin x \cos x \quad (y/r)$$

$$= \frac{2}{1}\left(\frac{-2}{\sqrt{5}}\right)\left(\frac{-1}{\sqrt{5}}\right)$$

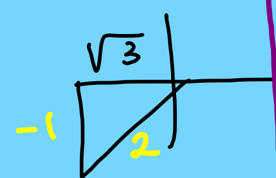
$$= \frac{4}{5}$$

$$1 + 4 = r^2$$

$$\sqrt{5} = \sqrt{r^2}$$

Find $\cos\left(\frac{A}{2}\right)$ given $\sin A = \frac{-1}{2} \frac{y}{r}$ & A in Q III.

$180° < A < 270°$
$90° < \frac{A}{2} < 135°$
Q II



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$$\frac{1+\cos A}{2} \quad \frac{A}{2} \text{ in Q II}$$

$$= -\sqrt{\frac{1+\sqrt{3}/2}{2}}$$

$$= \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{\frac{2}{1}}}$$

$$= -\sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}}$$

$$= -\sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= -\frac{\sqrt{2+\sqrt{3}}}{2}$$

$$1 + x^2 = 4$$

$$\sqrt{x^2} = \sqrt{3}$$

Verify.

$$\frac{\sin 2x}{1 - \cos 2x} = \frac{2\sin^2 (x/2) - 1}{-\sin x}$$

$$\frac{2 \sin x \cos x}{\cancel{1} - (\cancel{1} - 2\sin^2 x)} = \frac{\cancel{2}\left(\sqrt{\frac{1-\cos x}{\cancel{2}}}\right)^2 - 1}{- \sin x}$$

$$\frac{\cancel{2} \cancel{\sin x} \cos x}{2\sin^{\cancel{2}} x} = \frac{1 - \cos x - 1}{-\sin x}$$

$$\frac{\cos x}{\sin x} = \frac{\cancel{-}\cos x}{\cancel{-}\sin x}$$

Tips for choosing cos 2x:

1) Look at the opposite side
2) Choose the cos 2x identity that will make terms cancel.

62/ $\sin 4x = 4\sin x\cos x - 8\sin^3 x\cos x$

$\sin(2\cdot 2x) = 4\sin x\cos x\,(1 - 2\sin^2 x)$

$\sin(2\cdot A)$ $\quad 2\cdot \underbrace{2\sin x\cos x}_{\sin 2x}$

$2\sin 2x\cos 2x = 2\ \sin 2x \cdot \cos 2x$