FUNDAMENTAL IDENTITIES

Identity – true for any value you put in the variable.

$2(x+5) = 2x+10$

Trig Identities – True for any angle measure.

↳ change complicated expressions to a simpler form.

Reciprocal

1) $\csc\theta = \dfrac{1}{\sin\theta}$

$\sin\theta = \dfrac{1}{\csc\theta}$

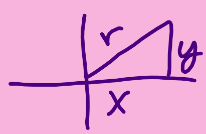
2) $\sec\theta = \dfrac{1}{\cos\theta}$

3) $\cot\theta = \dfrac{1}{\tan\theta}$

Ratio

4) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

5) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$



Pythagorean

6) $\sin^2\theta + \cos^2\theta = 1$

$\sin^2 30° + \cos^2 30° = 1$

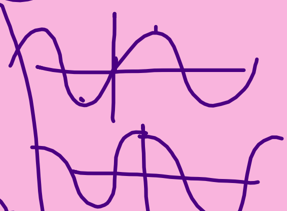
$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 =$

$\frac{1}{4} + \frac{3}{4} = 1$

7) $1 + \tan^2\theta = \sec^2\theta$

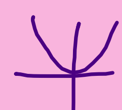
8) $1 + \cot^2\theta = \csc^2\theta$

$$\boxed{\begin{aligned} \sin(-\theta) &= -\sin\theta \\ \cos(-\theta) &= \cos\theta \\ \tan(-\theta) &= -\tan\theta \end{aligned}}$$



Even

$f(-x) = f(x)$



Odd

$f(-x) = -f(x)$

Simplify.

$$\frac{(1+\cos x)\cos x}{(1+\cos x)\sin x} + \frac{\sin x\,(\sin x)}{(1+\cos x)(\sin x)}$$

$$= \frac{\cos x + (\cancel{\cos^2 x + \sin^2 x})^{1}}{\sin x\,(1+\cos x)} \quad \#6$$

$$= \frac{\cancel{\cos x + 1}}{\sin x\,(\cancel{1+\cos x})}$$

$$= \frac{1}{\sin x} = \boxed{\csc x}$$

$$(1+\tan x)^2 - 2\tan x$$

$$(1+\tan x)(1+\tan x) - 2\tan x$$

$$1 + \cancel{2\tan x} + \tan^2 x - \cancel{2\tan x}$$

$$= 1 + \tan^2 x$$

$$= \boxed{\sec^2 x}$$

$$\frac{\sec^3 x - 8}{\sec^2 x - 4}$$

$$\frac{x^3 - 8}{x^2 - 4} = \frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)}$$

$$\frac{\cancel{(\sec x - 2)}(\sec^2 x + 2\sec x + 4)}{\cancel{(\sec x - 2)}(\sec x + 2)}$$

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Verify.

$$\tan^2\theta\left(\frac{1}{\sec^2\theta}\right) \mp \cot\theta \tan(+\theta) = -\cos^2\theta$$

$$\frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \cancel{\cos^2\theta} - \left(\frac{1}{\cancel{\tan\theta}}\right)\cancel{\tan\theta} = -\cos^2\theta$$

$$\sin^2\theta - 1 = -\cos^2\theta$$

$$-\cos^2\theta = -\cos^2\theta$$

$$\frac{\sec\theta}{\sin\theta} - \frac{\sec\theta}{\csc(\cancel{-}\theta)} = \frac{1}{\tan\theta}$$

$$\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{1}} - \frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}} = \cot\theta \quad \#3$$

$$\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta} - \frac{1}{\cos\theta}\cdot\frac{\sin\theta}{1} = \frac{\cos\theta}{\sin\theta} \quad \#5$$

$$\frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta\cdot\sin\theta}{\cos\theta\cdot\sin\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\frac{1 - \sin^2\theta}{\sin\theta\cos\theta} =$$

$$\frac{x^2}{x} \qquad \frac{\cos^{\cancel{2}}\theta}{\sin\theta\,\cancel{\cos\theta}} =$$

$$\frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\cot^4 B - \csc^4 B = 1 - 2\csc^2 B$$

$$\cancel{(\cot^2 B - \csc^2 B)}(\cot^2 B + \csc^2 B) =$$

$$(-1)(\cot^2 B + \csc^2 B) =$$

$$(-1)(\csc^2 B - 1 + \csc^2 B) = 1 - 2\csc^2 B$$

$$(-1)(2\csc^2 B - 1)$$

$$-2\csc^2 B + 1 = 1 - 2\csc^2 B$$

Big Powers (>2)

- Factor

$$x^4 - y^4$$

$$(x^2 + y^2)(x^2 - y^2)$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\cot^2\theta - \csc^2\theta = -1$$

$$1 + \cot^2 B = \csc^2 B$$

$$\cot^2 B = \csc^2 B - 1$$