

# LOGARITHMS

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1614

Exponential Form

$$y = b^x$$

$$b > 0, b \neq 1$$

$$y > 0, x \in \mathbb{R}$$

Domain:  $(-\infty, \infty)$

Range:  $(0, \infty)$

Logarithmic Form

$$x = \log_b y$$

$$b > 0, b \neq 1$$

$$x \in \mathbb{R}$$

$$y > 0$$

Domain:  $(0, \infty)$

Range:  $(-\infty, \infty)$

Any # part  
of the log  
must be +

of inverses  
exponential  
functions

$$y = b^x$$

$$x = \log_b y$$

$$y = \log_b x$$

$$y = \log_b x$$

Domain:  $(0, \infty)$

$$y = \log_7 (x-3)$$

Find domain.

Test Points!

$$\begin{array}{c} - & + \\ \hline & 3 \\ & \circ \\ & (3, \infty) \end{array}$$

$$y = \log_6 (4-x^2) \quad (2-x)(2+x)$$

Find domain

$$\begin{aligned} 4-x^2 &= 0 \\ \sqrt{4} &= \sqrt{x^2} \\ \pm 2 &= x \end{aligned}$$

$$\begin{array}{c} - & + & - \\ \hline & (0) & \\ -2 & 0 & 2 \\ & \circ & \\ & (-2, 2) \end{array}$$

Common Logs

$$\log_{10} x = \log x$$

Natural Logs

$$\log_e x = \ln x$$

$$y = 2^x$$

x	y
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$

$$y = \log_2 x$$

x	y
$2^0 = 1$	0
$2^1 = 2$	1
$2^2 = 4$	2
$2^3 = 8$	3

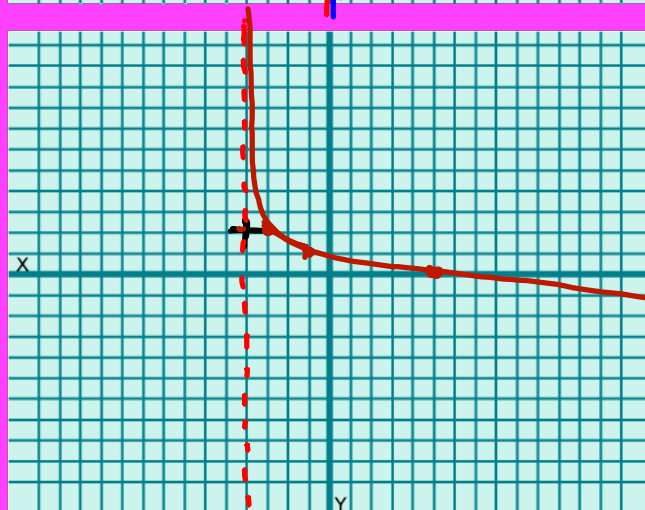
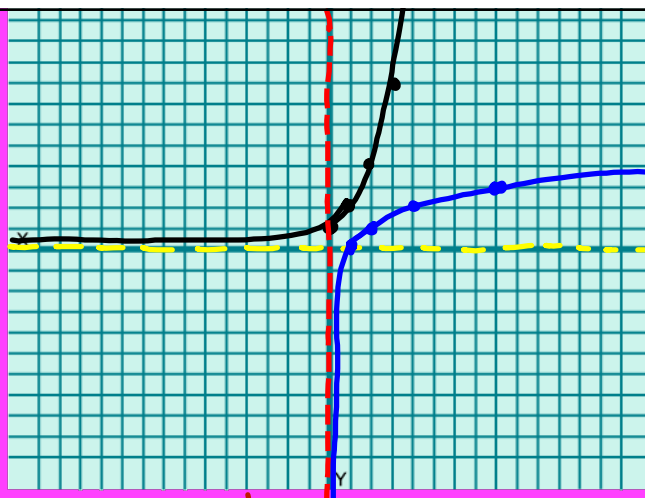
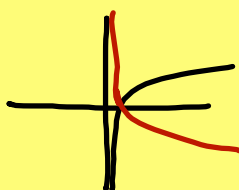
x-axis flip

$$y = -\log_3(x+4) + 2$$

left 4      up 2

$$y = 3^x$$

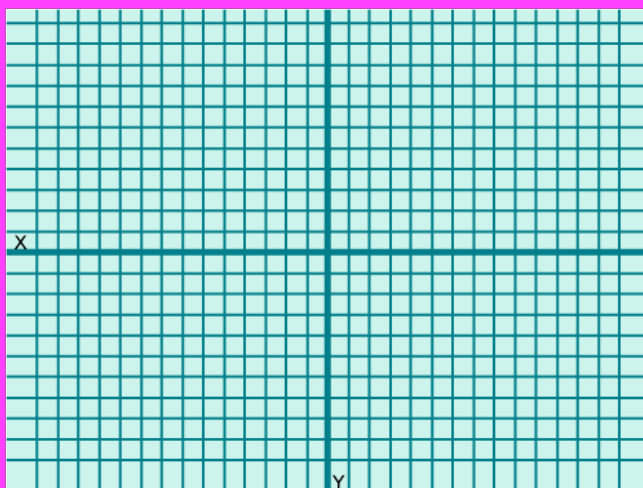
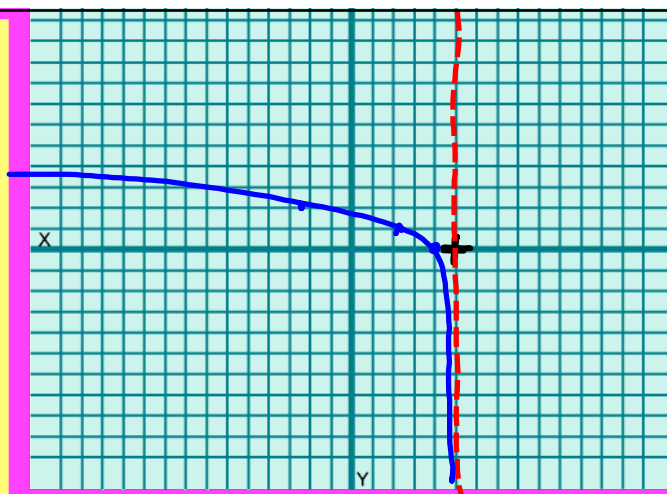
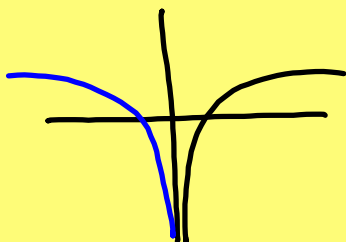
$3^0 = 1$	0
$3^1 = 3$	1
$3^2 = 9$	2
<del><math>3^3 = 27</math></del>	<del>3</del>



$$y = \ln_e(5-x) \quad \text{Right } 5$$

$e^0 = 1$	0
$e^1 = 2.7$	1
$e^2 = 7.4$	2

y-axis flip



# Evaluating Logs

$$2^3 = 8 \Leftrightarrow 3 = \log_2 8$$

$3 = \log_2 2^3$

$$\log_5 25 = 2 \Leftrightarrow 5^2 = 25$$

$$\log_5 5^{17} = 17$$

$$\log_2 4 = \log_2 2^2 = 2$$

$$\log_3 81 = \log_3 3^4 = 4$$

$$8^{\log_8 13} = 13$$

$$e^{\ln 123} = 123$$

argument

Logs represent  
exponents!

$$\log_{12} \frac{1}{144} = \log_{12} 12^{-2} = -2$$

$$\log_7 \sqrt[5]{49} = \log_7 \sqrt[5]{7^2} = \log_7 7^{2/5} = \frac{2}{5}$$

$$\log_{10} 1000 = \log_{10} 10^3 = 3$$

$$\ln e^{15} = 15$$

$$\ln \sqrt[7]{e^3} = \ln e^{3/7} = 3/7$$

# SOLVING LOGARITHMIC EQUATIONS

Solve for x.

$$\log_5 x = 4$$

$$5^{\log_5 x} = 5^4$$

$$\boxed{x = 625}$$

Check for + values!

EXPONENTIATE!

$$\log_a 64 = 2$$

$$a^{\log_a 64} = a^2$$

$$\sqrt{64} = \sqrt{a^2}$$

$$\pm 8 = a$$

$$\log_{25} \sqrt[4]{125} = x$$

$$25^{\log_{25} \sqrt[4]{125}} = 25^x$$

$$\sqrt[4]{125} = 25^x$$

$$\sqrt[4]{5^3} = 5^{2x}$$

$$5^{3/4} = 5^{2x}$$

$$\frac{1}{2} \cdot \frac{3}{4} = 2x \cdot \frac{1}{2}$$

$$\frac{3}{8} = x$$

Make  
Common  
bases!

No need to  
check - x  
was not  
part of  
the log