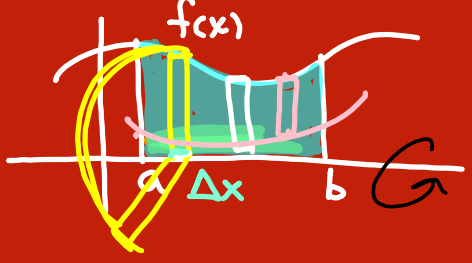
February 11, 2022

VOLUME OF SOLIDS OF REVOLUTION

DISK METHOD



$V = \pi r^2 h$ ← thickness

$\pi [f(x)]^2 \Delta x$

$\pi \int_a^b [f(x)]^2 dx$

$\pi r_o^2 \Delta x - \pi r_i^2 \Delta x$

$$\pi \int_a^b (r_o^2 - r_i^2) dx$$

▯ is ⊥ to axis of revolution

▯ is vertical Top-Bottom

▯ is horizontal R-L

$y = 2x \quad y = x^2$

Revolve around x-axis

$$\pi \int_a^b \left[(r_o^2) - (r_i)^2\right] dx$$

$$\pi \int_0^2 \left[(2x)^2 - (x^2)^2\right] dx$$

$$= \frac{64\pi}{15} \text{ units}^3$$

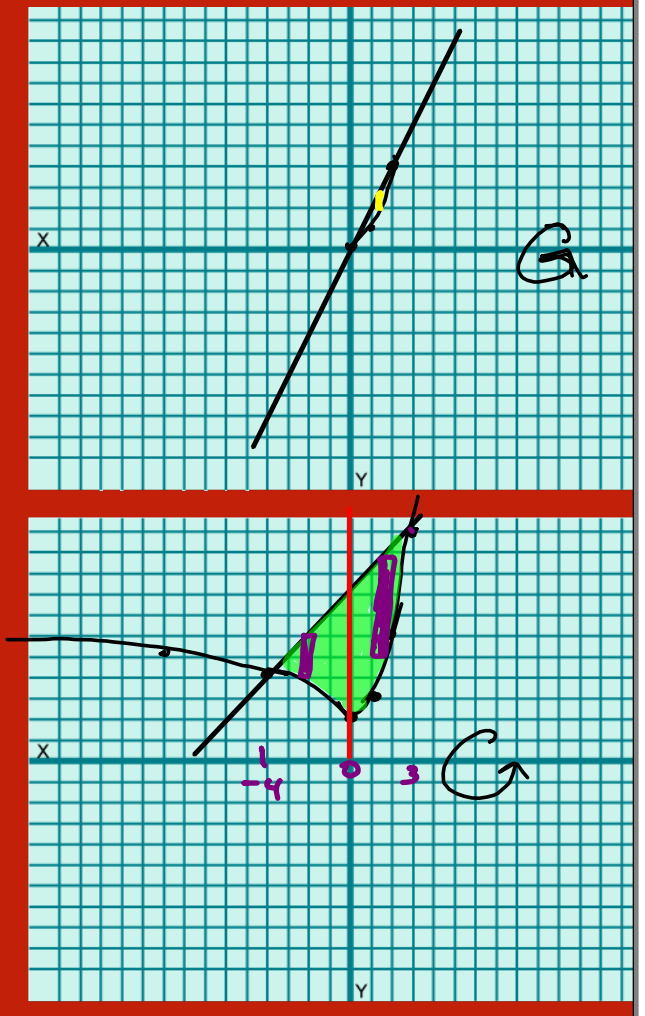
$y = x^2 + 2 \quad y = \sqrt{-x} + 2$

$y = x + 8$ around x-axis

0	0
-1	1
-4	2
-9	3

$$\pi \int_{-4}^{0} \left[(x+8)^2 - (\sqrt{-x}+2)^2\right] dx$$

$$+ \pi \int_0^3 \left[(x+8)^2 - (x^2+2)^2\right] dx$$



$y = x^2 + 4 \quad y = 2x^2 \quad x = 0$

only in 1st quad.

*around y-axis.

0	0
1	2
2	8
3	18

$y = x^2 + 4$

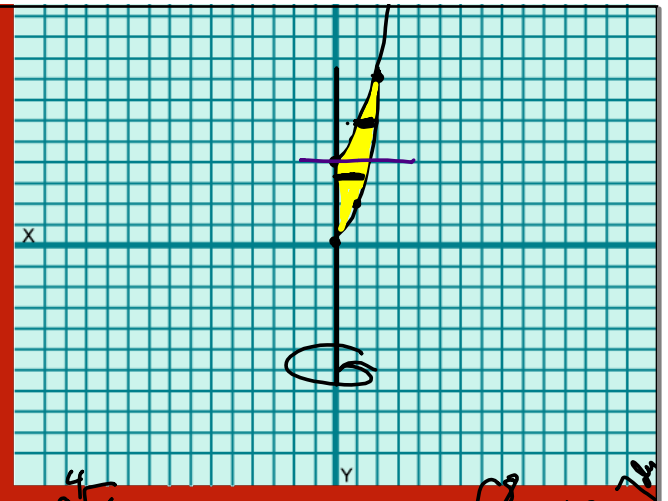
$\pm\sqrt{y-4} = \sqrt{x^2}$

$\sqrt{y-4} = x$

$y = 2x^2$

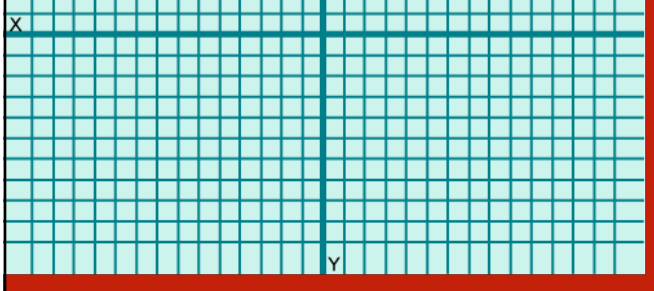
$\pm\sqrt{\frac{y}{2}} = \sqrt{x^2}$

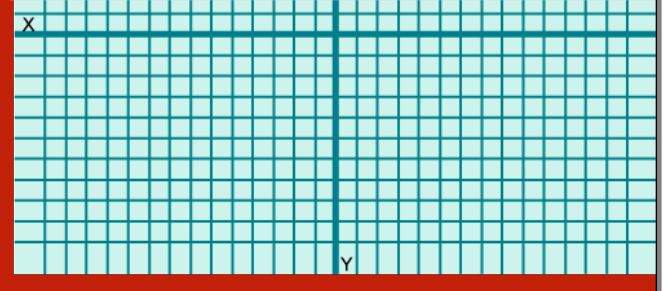
$\sqrt{\frac{y}{2}} = x$



$$\pi\int_0^4\left[\left(\sqrt{\frac{y}{2}}\right)^2 - 0^2\right]dy + \pi\int_4^8\left[\left(\sqrt{\frac{y}{2}}\right)^2 - \left(\sqrt{y-4}\right)^2\right]dy$$

$= 8\pi$ units^3





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