Business Appl. of Diff. Eq.

The marginal revenue for digital watches is expressed by $\int \frac{dR}{dx} = \int 60{,}000 - \frac{40000}{x^2}$ dollars per thousand. If total sales revenue is $38,000 when 1000 watches are sold, what will revenue be for 4000 watches?

$$R(x) = \int \left(60{,}000 - 40{,}000x^{-2}\right) dx$$

$$R(x) = 60{,}000x + \frac{40000x^{-1}}{-1} + C$$

$$R(x) = 60{,}000x + \frac{40000}{x} + C$$

$$38{,}000 = 60000 + 40000 + C$$

$$-100{,}000$$

$$-62{,}000 = C$$

$$R(x) = 60{,}000x + \frac{40000}{x} - 62{,}000$$

4000 watches

$$R(4) = 240{,}000 + 10{,}000 - 62{,}000$$

$$= \$188{,}000$$

(b) $C(x) = 2000x^2 + \frac{40000}{x} + 20{,}000$

How many watches sold to maximize profit?
Max production is 20,000 watches.

$P = R - C$ $\quad R(x) = 60{,}000x + \frac{40000}{x} - 62{,}000$

$P(x) = 60{,}000x + \frac{40000}{x} - 62000 + \left[2000x^2 + \frac{40000}{x} + 20{,}000\right]$

$P(x) = -2000x^2 + 60{,}000x - 82{,}000$

1) Find critical pt.

$P'(x) = -4000x + 60{,}000 = 0$

$60{,}000 = 4000x$

$15 = x$

$[0, 20]$

$P(0) = -82{,}000$

$P(15) = 368{,}000$

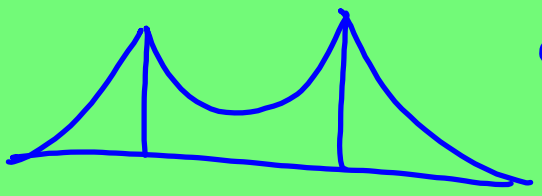
$P(20) = 318{,}000$

Produce 15,000 watches

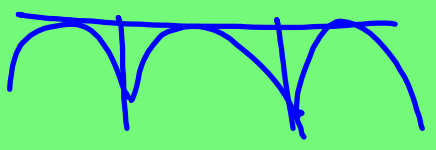
HYPERBOLIC FUNCTIONS

- Combinations of e^x & e^{-x}
- properties of trig functions
- connected through complex numbers

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$



Catenary $y = a\cosh\left(\frac{x}{a}\right) + c$



$$\sinh(\ln 3) = \frac{e^{\ln 3} - e^{-\ln 3^{-1}}}{2}$$

$$\frac{3 - 3^{-1}}{2} = \frac{8/3 - 1/3}{2} = \frac{2}{1}$$

$$= 4/3$$

Derivatives

$\frac{d}{dx}\sinh x = \cosh x$ $\quad$ $\frac{d}{dx}\cosh x = \sinh x$

$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$ $\quad$ $\frac{d}{dx}\coth x = -\operatorname{csch}^2 x$

$\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$ $\quad$ $\frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x$

$\cosh^2 x - \sinh^2 x = 1$

$f(x) = \coth x \cdot e^{\operatorname{csch}(x^3)}$ $\quad$ Find $f'(x)$

$f'(x) = \coth x \cdot e^{\operatorname{csch} x^3} \cdot -\operatorname{csch}(x^3)\coth(x^3) \cdot 3x^2 + e^{\operatorname{csch} x^3} \cdot -\operatorname{csch}^2 x$

$$\int \sinh^7 x \cosh x \, dx$$

$u = \sinh x$
$du = \cosh x \, dx$

$$\int u^7 \cancel{\cosh x} \cdot \frac{du}{\cancel{\cosh x}}$$

$$= \frac{u^8}{8} + C$$

$$= \boxed{\frac{\sinh^8 x}{8} + C}$$

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