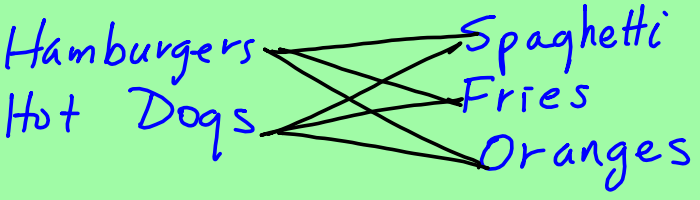
Combinatorics & Probability

of ways to perform an event



$2 \cdot 3 = 6$

Fundamental Counting Principle--If there are p ways to do one event and q ways to do another, then there are p•q ways to do both.

Car manufacturer

- 8 body colors
- 2 fabrics
- 3 option packages

$8 \cdot 2 \cdot 3 = 48$

How many different cars can be made?

Permutations – # of arrangements or patterns that can be formed from a set of objects

Linear Permutations

1) All Objects = $n!$

How many ways are there to arrange the students sitting in the front row?

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$= 5! = 5$ factorial

2) Arrange a small group chosen from a larger group $= {}_nP_r$

$9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$

${}_nP_r = \frac{n!}{(n-r)!}$

${}_nP_r$ ← # of items to use; n ← total # of items; P ← Permutations

$= {}_9P_5 = \frac{9!}{(9-5)!} = \frac{9!}{4!}$

$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}}$

${}_7P_2 = \frac{7!}{5!} = \frac{7 \cdot 6 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}$

$= 42$

How many ways can 4 relay runners be positioned for a race if chosen from 6 possible team members?

${}_nP_r = {}_6P_4 = 360$

3) Alike Objects--indistinguishable, identical $= \frac{\text{total}!}{\text{alike}!\ \text{alike}!}$

How many permutations of the letters in the word MISSISSIPPI are possible?

$$\frac{11!}{4!\,4!\,2!} = 34{,}650$$

S I P

How many ways can 5 identical basketballs, 4 identical volleyballs, and 3 identical soccer balls be arranged in a line?

$$\frac{12!}{5!\,4!\,3!} = 27{,}720$$

4) Specific locations or Repeated objects – Draw blanks

How many ways can six people from this class be arranged in row if there must be a sophomore on each end of the row and two juniors in the middle seats?

12 students
1 sr
2 jr.
9 so.

$9 \cdot 8 \cdot 2 \cdot 1 \cdot 7 \cdot 8 = 8064$

How many different license plates are possible with 3 letters followed by 3 digits letters cannot be repeated but digits can be repeated?

$\underset{L}{26} \cdot \underset{L}{25} \cdot \underset{L}{24} \cdot \underset{\#}{10} \cdot \underset{\#}{10} \cdot \underset{\#}{10} = 15{,}600{,}000$

COMBINATIONS – the # of groups that can be formed from a set of objects

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

$$_7C_5 = \frac{7!}{2!\,5!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 21$$

Mrs. Meyer wants to form an "I Love Math" Committee to promote mathematics throughout the school. How many different committees of 5 students can be selected from this class?

12 students

$$_{12}C_5 = 792$$

Menu - Probability - #3 Combinations

Card Facts

52 cards (no jokers)
26 red, 26 black
4 suits – ♡, ◇, ♧, ☆
13 cards in a suit
4 cards of each type
12 face cards

Draw 5 cards. How many hands of 5 diamonds are possible?

$_{13}C_5 = 1287$

How many hands with a full house are possible?

3 of a Kind + 2 of a Kind

$_{13}C_1 \cdot {}_4C_3 \cdot {}_{12}C_1 \cdot {}_4C_2$

$= 3744$

How many hands with 3 black cards and 2 red cards are possible?

multiply

$_{26}C_3 \cdot {}_{26}C_2$

$= 845{,}000$

$_{52}C_5 = 2{,}598{,}960$