APPLICATIONS OF INTEGRATION

Differential Equations

Find general solution.

$$\int \frac{d^2y}{dx^2} = \int 24x^2 + 18x + 4$$

$$\frac{dy}{dx} = \frac{24x^3}{3} + \frac{18x^2}{2} + 4x + C_1$$

$$\int \frac{dy}{dx} = \int 8x^3 + 9x^2 + 4x + C_1$$

$$\boxed{y = 2x^4 + 3x^3 + 2x^2 + C_1x + C_2}$$

complete/general solution
- has + C

particular solution
solve for C.

Find particular solution.

$y = -1$ when $x = 0$

$\Rightarrow y' = 9$ when $x = 2$

$$\int \frac{d^2y}{dx^2} = \int 3x^2$$

$$\frac{dy}{dx} = x^3 + C$$

$$9 = (2)^3 + C$$

$$9 = 8 + C$$

$$1 = C$$

$$\int \frac{dy}{dx} = \int x^3 + 1$$

$$y = \frac{1}{4}x^4 + x + C$$

$$-1 = 0 + 0 + C$$

$$\boxed{y = \frac{1}{4}x^4 + x - 1}$$

Motion

$s(t) \leftarrow$ position

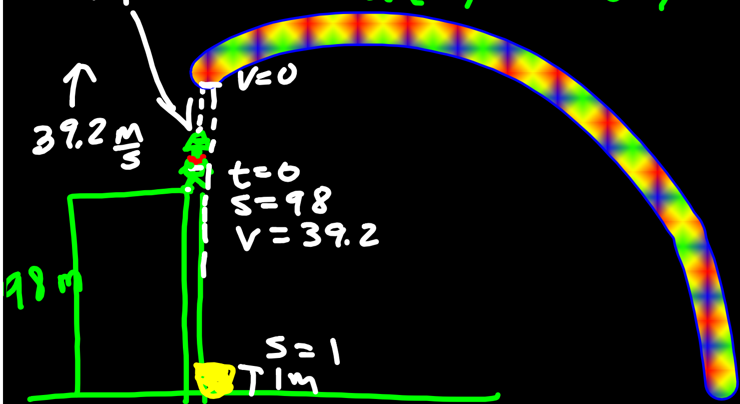
$v(t) = s'(\cancel{t})$

$a(t) = v'(t) = s''(t)$

$a = -9.8\ m/s^2$

$a = -32\ ft/s^2$

leprechaun



$a(t) = -9.8$

$v(t) = -9.8t + C$

$39.2 = \quad 0 + C$

$\boxed{v(t) = -9.8t + 39.2}$

$s(t) = -4.9t^2 + 39.2t + C$

$98 = \quad 0 + 0 + C$

$\boxed{s(t) = -4.9t^2 + 39.2t + 98}$

What is maximum height?

$0 = -9.8t + 39.2$

$\frac{9.8t}{9.8} = \frac{39.2}{9.8}$

$t = 4$ sec

$s(4) = -4.9(4)^2 + 39.2(4) + 98$

$= 176.4$ m

How fast will he be moving when he reaches the pot of gold.

$1 = -4.9t^2 + 39.2t + 98$

$0 = -4.9t^2 + 39.2t + 97$

↑ quadr. formula

$0 = -4.9(t^2 - 8t - 20)$

$-4.9\ (t - 10)(t + 2)$

$t = 10, \cancel{-2}$

10 sec.

$v(10) = -9.8(10) + 39.2$

$= -58.8\ m/s \approx 131.53\ \frac{mi}{h}$

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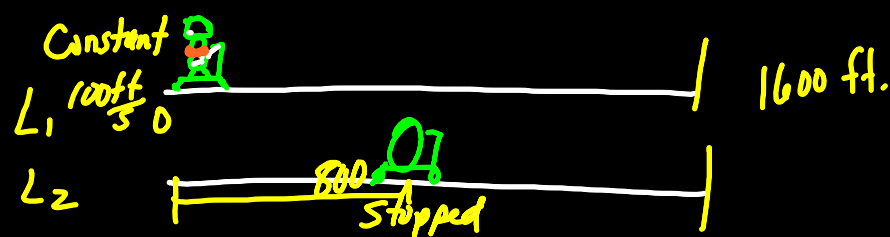
Will L_1 catch L_2 before the finish line.



$a = 10\ \frac{ft}{s^2}$ after 2 sec $30\ \frac{ft}{sec}$

L_1

$a(t) = 0$

$v(t) = 25$

$s(t) = 25t + c$

$0 = 25(0) + c$

$s(t) = 100t$

L_2

$a(t) = 10$

$v(t) = 10t + c$

$30 = 10(2) + c$

$30 = 20 + c$

$10 = c$

$v(t) = 10t + 10$

$s(t) = 5t^2 + 10t + c$

$800 = 0 + 0 + c$

$s(t) = 5t^2 + 10t + 800$

$100t = 5t^2 + 10t + 800$

$0 = 5t^2 - 90t + 800$

$0 = 5(t^2 - 18t + 160)$

pretend $t = 8.8$