Operations in Polar Form

$$2(\cos 30^\circ + i\sin 30^\circ)\cdot 5(\cos 70^\circ + i\sin 70^\circ)$$

$$10(\cos 30^\circ\cos 70^\circ + i\cos 30^\circ\sin 70^\circ + i\sin 30^\circ\cos 70^\circ \mp \cancel{i}\sin 30^\circ\sin 70^\circ)$$

$$10[\cos(30^\circ+70^\circ) + i\sin(30^\circ+70^\circ)]$$

$$= 10[\cos 100^\circ + i\sin 100^\circ]$$

$$r_1(\cos\theta_1 + i\sin\theta_1)\cdot r_2(\cos\theta_2 + i\sin\theta_2) =$$

$$r_1 r_2\,(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

$$37(\cos 211^\circ + i\sin 211^\circ)\cdot 4(\cos 348^\circ + i\sin 348^\circ)$$

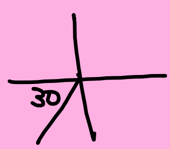
$$= \boxed{148(\cos \overset{211+348}{559^\circ} + i\sin 559^\circ)}$$

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}\left(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right)$$

Divide & change to rectangular form.

$$\frac{15(\cos 340^\circ + i\sin 340^\circ)}{3(\cos 550^\circ + i\sin 550^\circ)} = 5\left(\cos(340^\circ-550^\circ) + i\sin(340^\circ-550^\circ)\right)$$

$$= 5\left[\cos(+210^\circ) \mp i\sin(+210^\circ)\right]$$



$$= 5\left[-\frac{\sqrt{3}}{2} + i\left(+\frac{1}{2}\right)\right]$$

$$= -\frac{5\sqrt{3}}{2} + \frac{5}{2}i$$

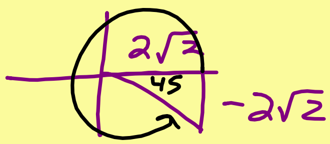
DeMoivre's Theorem

$$\left[r(\cos\theta + i\sin\theta)\right]^3 = r^3(\cos 3\theta + i\sin 3\theta)$$

*Journal $$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n\left[\cos(n\cdot\theta) + i\sin(n\cdot\theta)\right]$$

$$\left(2\sqrt{2} - 2i\sqrt{2}\right)^6$$

1) Change to polar form
2) Use DeMoivre's Thm to do 6th power
3) Convert back to rectangular



$$(2\sqrt{2})^2 + (-2\sqrt{2})^2 = r^2$$
$$8 + 8 = r^2$$
$$\sqrt{16} = \sqrt{r^2}$$
$$4 = r$$

$$\tan\theta = \frac{-2\sqrt{2}}{2\sqrt{2}} = -1$$
$$\theta = 315°$$

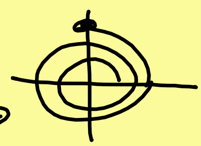
$$\left[4(\cos 315° + i\sin 315°)\right]^6$$

$$4096(\cos \cancel{1890°} + i\sin 1890°)$$ $315°\cdot 6$

$$4096(0 + i(1))$$

$$= 0 + 4096i$$

90°



$$\frac{1890}{360} = 5.25$$

When is polar form better? 1) complex # raised to a power

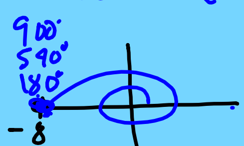
$(x^4)^{1/4} = (1236)^{1/4}$ 2) finding roots of a complex #

Solve.

$x^3 + 8 = 0$

$(x^3)^{1/3} = (-8)^{1/3}$

$x^3 = (-8 + 0i)^{1/3}$



$r = 8$

$\theta = 180°$

$360° \cdot \frac{1}{3} = 120°$

1) $\left[8(\cos \underline{180°} + i \sin \underline{180°})\right]^{1/3}$

$8^{1/3}$ $\quad 180° \cdot 1/3$

$= 2(\cos 60° + i \sin 60°)$

$2(\,1/2 + i \frac{\sqrt{3}}{2}) = \boxed{1 + i\sqrt{3}}$

2) $\left[8(\cos 540° + i \sin 540°)\right]^{1/3}$

$540° \cdot 1/3$

$2(\cos 180° + i \sin 180°)$

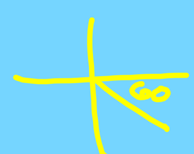
$2(-1 + 0) = \boxed{-2}$

3) $\left[8(\cos 900° + i \sin 900°)\right]^{1/3}$

$= 2(\cos 300° + i \sin 300°)$

$= 2(1/2 + i \frac{\sqrt{3}}{2})$

$= \boxed{1 - i\sqrt{3}}$



1) Isolate the variable.

2) Eliminate the power on the variable by using the 1/n power.

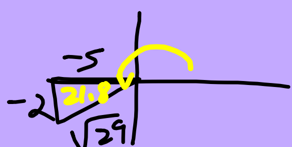
3) Change to polar form.

4) Apply DeMoivre's Theorem.

5) Get additional answers by taking $(1/n) \cdot 360^{\circ}$ and add to first answer.

$x^4 - (-5-2i) = 0$

$(x^4)^{1/4} = (-5-2i)^{1/4}$



$\tan\theta = \frac{+2}{+5}$

$\theta = 201.8°$

Find the 4^{th} roots of $(-5-2i)$

$\left[\sqrt{29}\,(\cos 201.8° + i\sin 201.8°)\right]^{\frac{1}{4}}$

$(\sqrt{29})^{1/4} = (29^{1/2})^{1/4}$

$201.8° \cdot \frac{1}{4}$

$360° \cdot \frac{1}{4} = 90°$

1) $29^{1/8}(\cos 50.45° + i\sin 50.45°)$

2) $29^{1/8}\ \text{cis}\ 140.45°$

3) $29^{1/8}\ \text{cis}\ 230.45°$

4) $29^{1/8}\ \text{cis}\ 320.45°$