WORK

Work = Force · distance

= Newtons · m = N m = joules

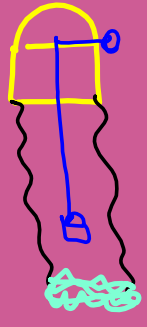
= lbs. · ft = ft·lb

20 lb of force / moved 10 ft.

$W = 20\text{ lb} \cdot 10\text{ ft} = 200\text{ ft. lb.}$

Calculus – Force can vary

$$W = \int_0^{50} F(x)\,dx$$



Well = 40 ft deep

Bucket weighs 30 lb. when full

Loses $\frac{1}{4}$ lb for each ft. it is raised.

How much work is performed

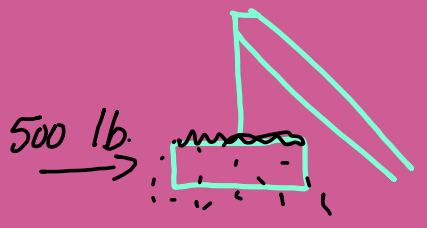
$$W = \int_0^{40} \left(30 - \frac{1}{4}x\right) dx = 1000\text{ ft·lb}$$

Crane raises bucket of sand

Raises 50 ft in 10 sec

Sand spills out 10 lb/sec.

Cable weighs 5 lb/ft.



How much work is needed to raise it 13.5 ft?

$$\int_0^{13.5} \left[\underbrace{(500 - 2x)}_{\text{bucket}} + \underbrace{5x}_{\text{cable}} \right] dx$$

$10 \frac{\text{lb}}{\cancel{\text{sec}}} \cdot \frac{10 \cancel{\text{sec}}}{50 \text{ ft.}} = 2 \frac{\text{lb}}{\text{ft.}}$

$$\int_0^{13.5} (500 + 3x)\, dx = 9487 \text{ ft} \cdot \text{lb.}$$

1 hp = 550 ft·lb

SPRINGS

Hooke's Law

$F(x) = kx$

- k: spring constant
- x: distance stretched or compressed

A spring with natural length of 1 m requires a force of 8 N to stretch it 3 m. How much work is required to stretch it from a length of 2m to length of 4 m?

$F = kx$

$8 = k \cdot 3$

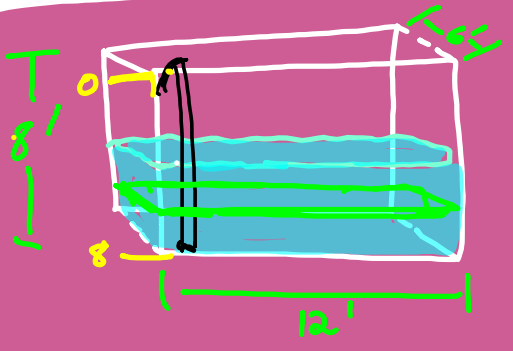
$\frac{8}{3} = k$

$F = \frac{8}{3}x$

$$\int_1^3 \frac{8}{3}x\,dx = \frac{32}{3}\ \text{J}$$

1 ← measured from natural length

Pump Problems



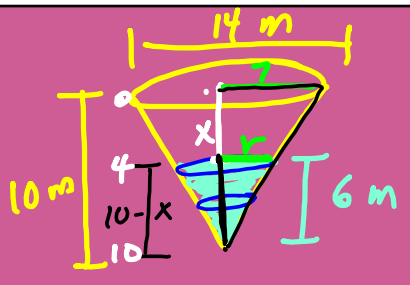
Weight density = ρ

$\rho = 62.4 \frac{lb}{ft^3}$ or $9810 \frac{N}{m^3}$

$$\int \rho \underbrace{A\,dx}_{\text{volume}} \cdot \text{depth}$$

$$\int_0^8 62.4 \cdot \underbrace{(6 \cdot 12)}_{w \cdot l} \cdot x \; dx$$

$$\approx 143{,}208 \text{ ft} \cdot \text{lb}$$



$$A = \pi r^2$$

$$\frac{r}{7} = \frac{10-x}{10}$$

$$r = \frac{7}{10}(10-x)$$

$$\int \rho \cdot A(x) \cdot \text{depth}\, dx$$

$$\int_4^{10} 9810 \cdot \pi \left[\frac{7}{10}(10-x)\right]^2 x \, dx$$

$$\approx 5,980,123.4 \text{ J}$$