SEQUENCES & SERIES

Sequence – a list of numbers that follow a pattern

Series – the sum of the numbers in a sequence

1, 2, 3, 4, ... Arith
+1

2, 4, 6, 8, ... Arith
+2

1, 2, 4, 7, 11,
+1 +2 +3 +4

5, -15, 45, -135, . . . Geom.
×-3

FIBONACCI SEQUENCE

1, 1, 2, 3, 5, 8, 13, 21, . . . a_n
a_1 a_2 a_3

a_n ↑ last term or unknown term

Leonardo de Fibonnacci

Nature =

n = # of terms

S_n = Sum of terms

Arithmetic sequences – add the same value to every term

d = common difference

Geometric sequences – multiply the same value to each term

r = common ratio

Find the first 4 terms.

$a_n = 4n + 2$

$n=1$ $a_1 = 4(1) + 2 = 6$

$a_2 = 4(2) + 2 = 10$

$a_3 = 4(3) + 2 = 14$

$a_4 = 4(4) + 2 = 18$

Add 4 = Arithmetic

$a_n = \frac{n+2}{2n}$

$a_1 = \frac{1+2}{2(1)} = \frac{3}{2}$

$a_2 = \frac{2+2}{2(2)} = \frac{4}{4} = 1$

$a_3 = \frac{3+2}{2(3)} = \frac{5}{6}$

$a_4 = \frac{4+2}{2(4)} = \frac{6}{8} = \frac{3}{4}$

SUMMATION NOTATION

Sigma

$$\sum_{n=1}^{4}(2n-3) = [2(1)-3] + [2(2)-3] + [2(3)-3] + [2(4)-3]$$

$$= -1 + 1 + 3 + 5$$

$$= \boxed{8}$$

$$\sum_{i=1}^{n}(p i + q)$$

ARITHMETIC

Sum $\Rightarrow$ $\sum_{j=22}^{50}(4j+7)$ (a_1 at $j=22$, a_n at $j=50$)

$n = 50-22+1 = 29$

$a_1 = 4(22)+7 = 95$

$a_n = 4(50)+7 = 207$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$= \frac{29}{2}(95+207)$$

$$= \boxed{4379}$$

Arithmetic Sequences

– adds the same value to each term.

d = common difference $= a_2 - a_1$

1, 2, 3, 4, ...

2, 4, 6, 8, ... $d = 2$

47, 42, 37, 32, ...

$d = -5$

3, 11, 19, 27, ...

a_2 a_3 a_4

$3+8$ $3+16$ $3+24$

$3+8\cdot1$ $3+8\cdot2$ $3+8\cdot3$

$d = 8$

Find the 200^{th} term.

$a_{200} = 3 + 8(199)$

$= 1595$

$a_n = a_1 + d(n-1)$

$\frac{17}{12}, \frac{5}{6}, \frac{1}{4}, \ldots$

$\frac{17}{12}, \frac{10}{12}, \frac{3}{12}, \ldots$

$d = \frac{-7}{12}$

Find 8^{th} term

$a_8 = \frac{17}{12} + \frac{-7}{12}(8-1)$

$a_9 = -\frac{8}{3}$

Arithmetic Series

← Sum of the terms

$S_n = 5 + 8 + 11 + 14$

$+ \quad S_n = 14 + 11 + 8 + 5$

$2S_n = 19 + 19 + 19 + 19$

$= 4 \cdot (19)$

$\frac{2S_n}{2} = \frac{76}{2}$

$S_n = 38$

$= 38$

$S_n = \frac{n}{2}(a_1 + a_n)$

Find S_n.

$52 + 64 + 76 + \cdots + 1816.$

$d = 64 - 52 = 12$

$a_n = a_1 + d(n-1)$

$1816 = 52 + 12(n-1)$

$-52 \quad -52$

$\frac{1764}{12} = \frac{12(n-1)}{12}$

$147 = n - 1$

$148 = n$

$S_n = \frac{148}{2}(52 + 1816)$

$= 138,232$

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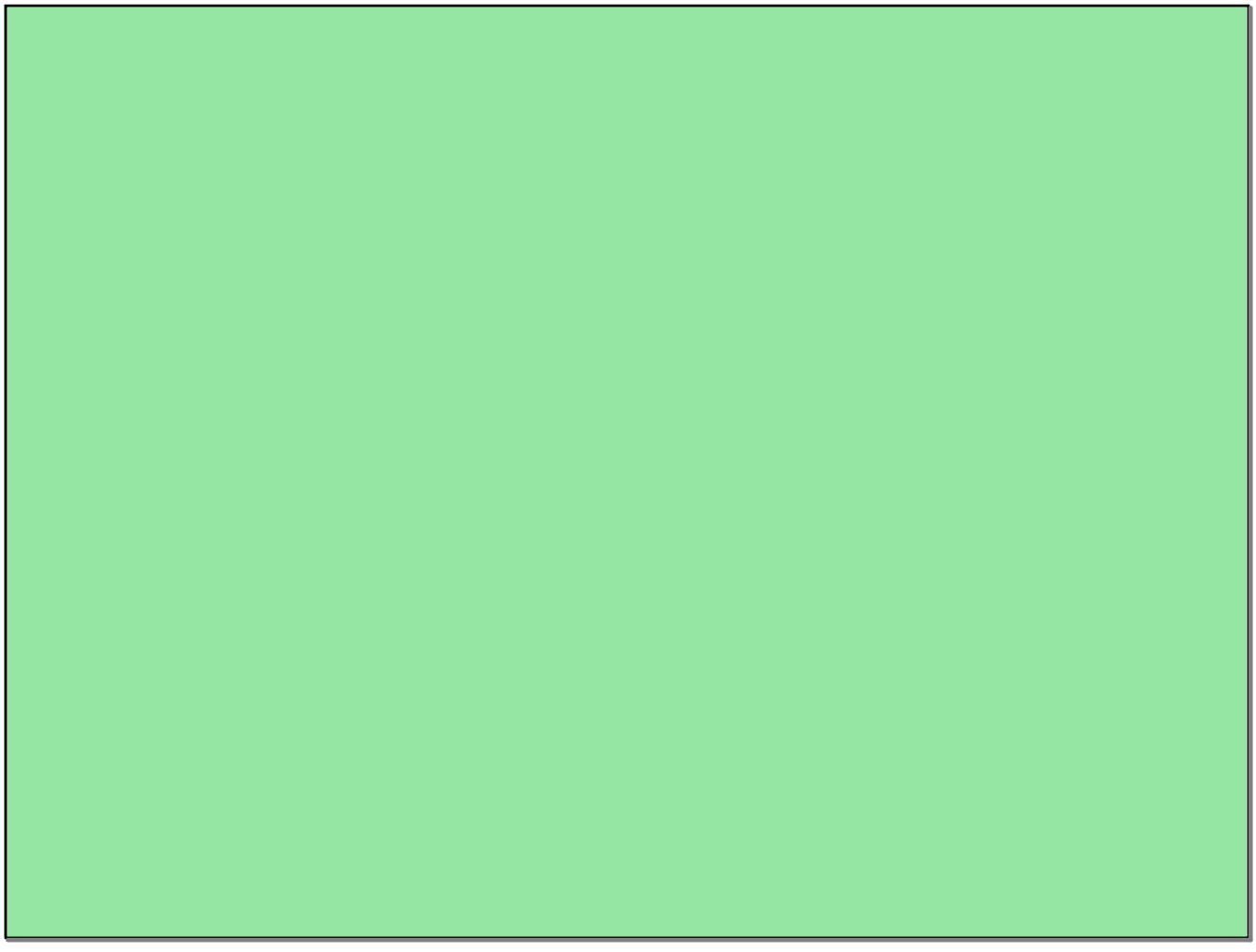

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