# TECHNIQUES OF INTEGRATION

Integration by parts — integrate two unrelated functions.

$$\int (f\cdot g)' = \int f\cdot g' + \int g\cdot f'$$

$$\int x^2 e^x \, dx$$

$$\int f\cdot g' \, dx$$

$$\int (f\cdot g)' - \int g\cdot f' = \int f\cdot g'$$

$$f\cdot g - \int g\cdot f' = \int f\cdot g'$$

$$\boxed{u\cdot v - \int v\cdot du = \int u\cdot dv}$$

$$\int x \sec^2 x \, dx$$

$$\int \underline{u} \cdot \underline{dv} = uv - \int v \, du$$

$u = x \qquad dv = \sec^2 x \, dx$

$du = 1 dx \qquad v = \tan x$

$$= x \tan x - \int \tan x \, dx$$

$$= x \tan x - \int \frac{\sin x}{\cos x} dx$$

$u = \cos x$

$du = -\sin x \, dx$

$\dfrac{du}{-\sin x} = dx$

$$= x \tan x + \int \frac{\cancel{\sin x}}{u} \frac{du}{+\cancel{\sin x}}$$

$$= x \tan x + \int \frac{1}{u} du$$

$$= x \tan x - \ln|u| + C$$

$$= \boxed{x \tan x - \ln|\cos x| + C}$$

$$\int \ln x \, dx$$

$u = \ln x$ $\quad \int dv = \int dx$

$du = \frac{1}{x} dx$ $\quad v = x$

$$= x \ln x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} \, dx$$

$$= x \ln x - \int 1 \, dx$$

$$= \boxed{x \ln x - x + C}$$

$$\int \ln(2x)$$

$u = 2x$

$du = 2\,dx$

$$\int \ln u \frac{du}{2}$$

$$\frac{1}{2}\left[u \ln u - u\right] + C$$

$$\frac{1}{2}\left[2x \ln 2x - 2x\right] + C$$

$$x \ln 2x - x + C$$

$$\int x^2 e^{2x}\,dx$$

$u = x^2$

$du = 2x\,dx$

$\int dv = \int e^{2x}\,dx$

$v = \int e^u \cdot \frac{du}{2}$

$u = 2x$

$du = 2\,dx$

$v = \frac{1}{2}e^u$

$v = \frac{1}{2}e^{2x}$

$$= \frac{1}{2}x^2 e^{2x} - \int x e^{2x}\,dx$$

$$= \frac{1}{2}x^2 e^{2x} + \left[-\frac{1}{2}x e^{2x} + \int \frac{1}{2}e^{2x}\,dx\right.$$

$u = x$

$du = dx$

$dv = e^{2x}\,dx$

$v = \frac{1}{2}e^{2x}$

$$= \boxed{\frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + C}$$

$$\int e^x \cos x\,dx$$

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x\,dx \qquad v = e^x$

$$= e^x \cos x + \int + e^x \sin x\,dx$$

$u = \sin x \qquad dv = e^x dx$

$du = \cos x\,dx \qquad v = e^x$

$$\int e^x \cos x\,dx = e^x \cos x + e^x \sin x - \int e^x \cos x\,dx$$

$$2\int e^x \cos x\,dx = e^x \cos x + e^x \sin x$$

$$\int e^x \cos x\,dx = \frac{1}{2}\left[e^x \cos x + e^x \sin x\right]$$

$$= \frac{1}{2} e^x \left[\cos x + \sin x\right]$$

deriv start with co- get –

Integ = answer is co-, get neg.