PARTIAL FRACTIONS

$\frac{Ax+B}{x^2+4}$

$$\int \frac{15x-14}{x^2-3x+2}\,dx = \frac{A}{x-2} + \frac{B}{x-1}$$

$(x-2)(x-1)$

$$15x-14 = A(x-1) + B(x-2)$$
$$15x-14 = Ax - A + Bx - 2B$$

$$15 = A + B$$
$$-14 = -A - 2B$$

$$\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 15 \\ -14 \end{bmatrix} = \begin{bmatrix} 16 \\ -1 \end{bmatrix} \begin{matrix} A \\ B \end{matrix}$$

$u = x-2$, $du = dx$

$$\int \frac{16}{x-2}\,dx + \int \frac{-1}{x-1}\,dx$$

$u = x-1$, $du = dx$

$$\int \frac{16}{u}\,du \qquad \int \frac{-1}{u}\,du$$

$$16\int \frac{1}{u}\,du$$

$16\ln|u|$ $\quad -\ln|u|$

$16\ln|x-2|$ $\quad -\ln|x-1| + C$

16

$$\ln\frac{|x-2|^{16}}{|x-1|} + C$$

$$\left[\int \frac{1-x^2}{9x^4+17x^2+4}\,dx = \frac{Ax+B}{4x^2+1} + \frac{Cx+D}{x^2+4}\right] \quad (4x^2+1)(x^2+4)$$

$$(4x^2+1)(x^2+4)$$

$$1-x^2 = (Ax+B)(x^2+4) + (Cx+D)(4x^2+1)$$

$$1-x^2 = Ax^3+4Ax+Bx^2+4B+4Cx^3+Cx+4Dx^2+D$$

$$0 = A \quad +4C$$
$$-1 = \quad B \quad +4D$$
$$0 = 4A \quad +C$$
$$1 = \quad 4B \quad +D$$

$$\begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 4 \\ 4 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/3 \\ 0 \\ -1/3 \end{bmatrix}$$

$$\int \frac{\frac{1}{3}}{4x^2+1}\,dx + \int \frac{-1/3}{x^2+4}\,dx$$

$$\frac{1}{3}\int \frac{1}{4x^2+1} \quad u=2x \quad du=2dx \quad -\frac{1}{12}\int \frac{1}{\frac{1}{4}x^2+1}\,2\cdot du \quad \begin{cases} u=\frac{1}{2}x \\ du=\frac{1}{2}dx \end{cases}$$

$$\frac{1}{3}\int \frac{1}{u^2+1}\,\frac{du}{2} \quad -\frac{1}{6}\int \frac{1}{u^2+1}\,du \qquad (2x)^2$$

$$= \frac{1}{6}\tan^{-1}u \quad -\frac{1}{6}\tan^{-1}u$$

$$\frac{1}{6}\tan^{-1}(2x) - \frac{1}{6}\tan^{-1}\left(\frac{1}{2}x\right) + C$$

$$\frac{\sim\sim}{(x-4)^2 \cdot x^3} = \frac{A}{(x-4)^2} + \frac{B}{(x-4)^1} + \frac{C}{x^3} + \frac{D}{x^2} + \frac{E}{x}$$

$$\frac{1}{x} + \frac{1}{x^3}$$

$$\int \frac{x^3 - 2x^2 + 1}{x^2 - 4x}$$

← Do long division & then do partial fractions on the remainder.

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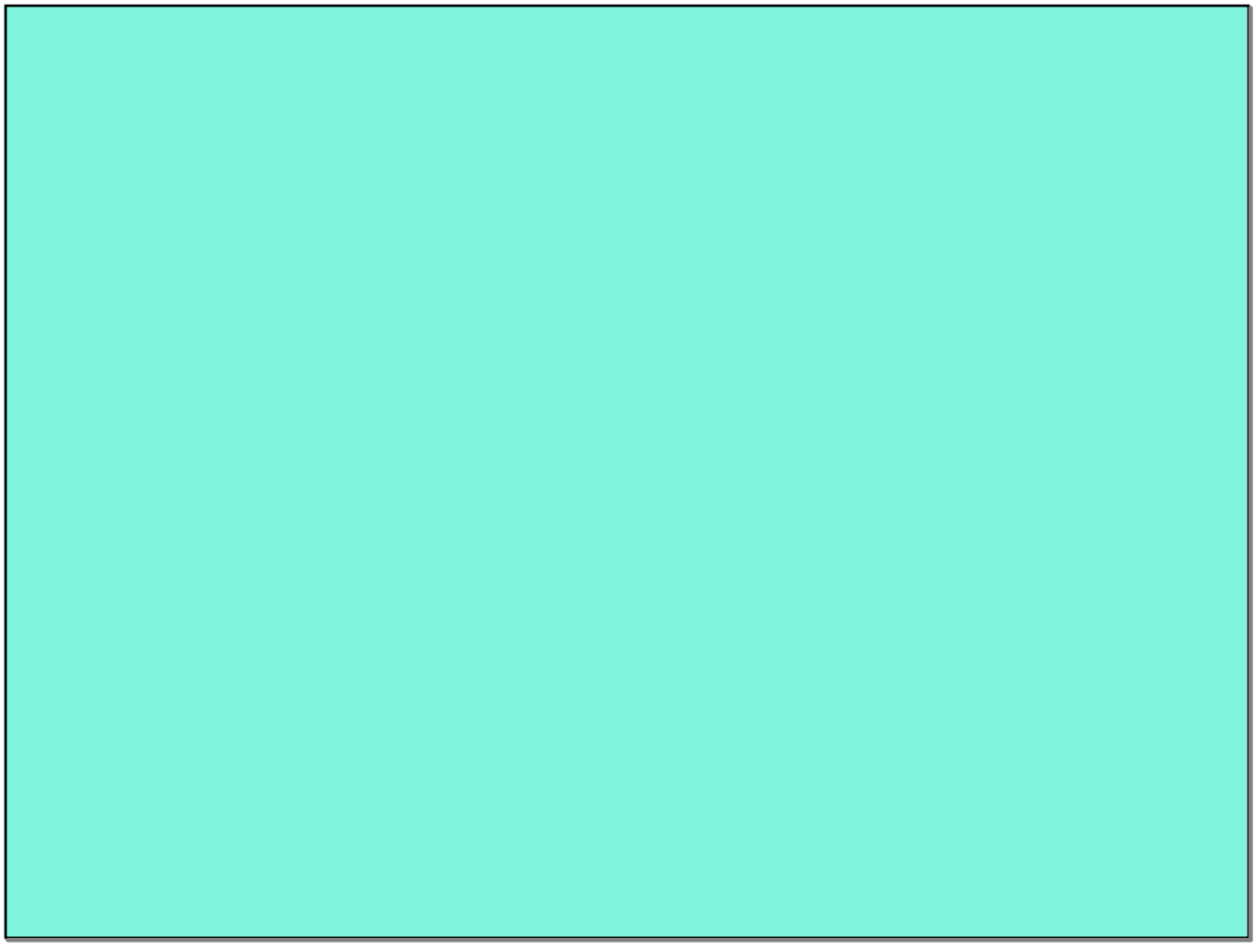

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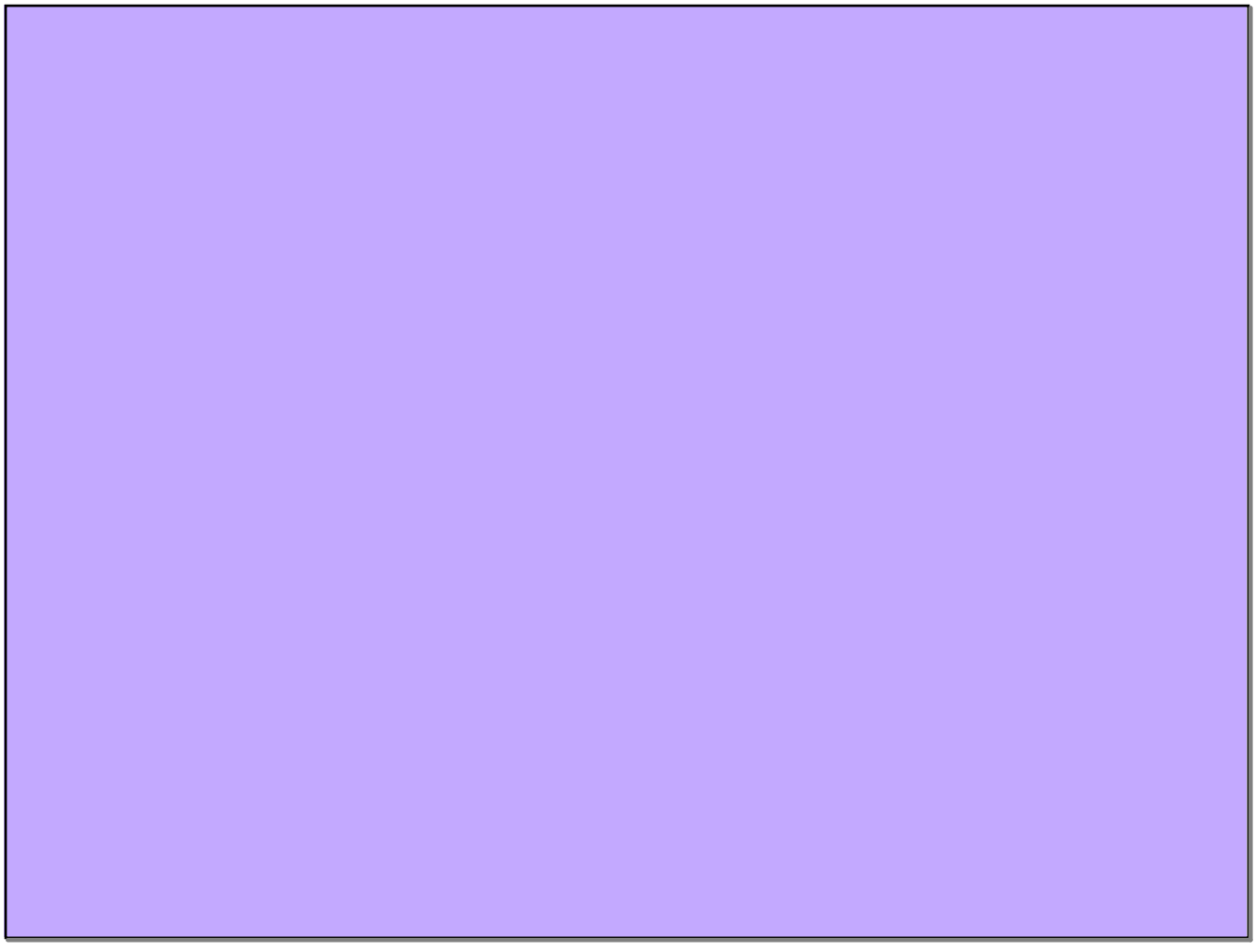

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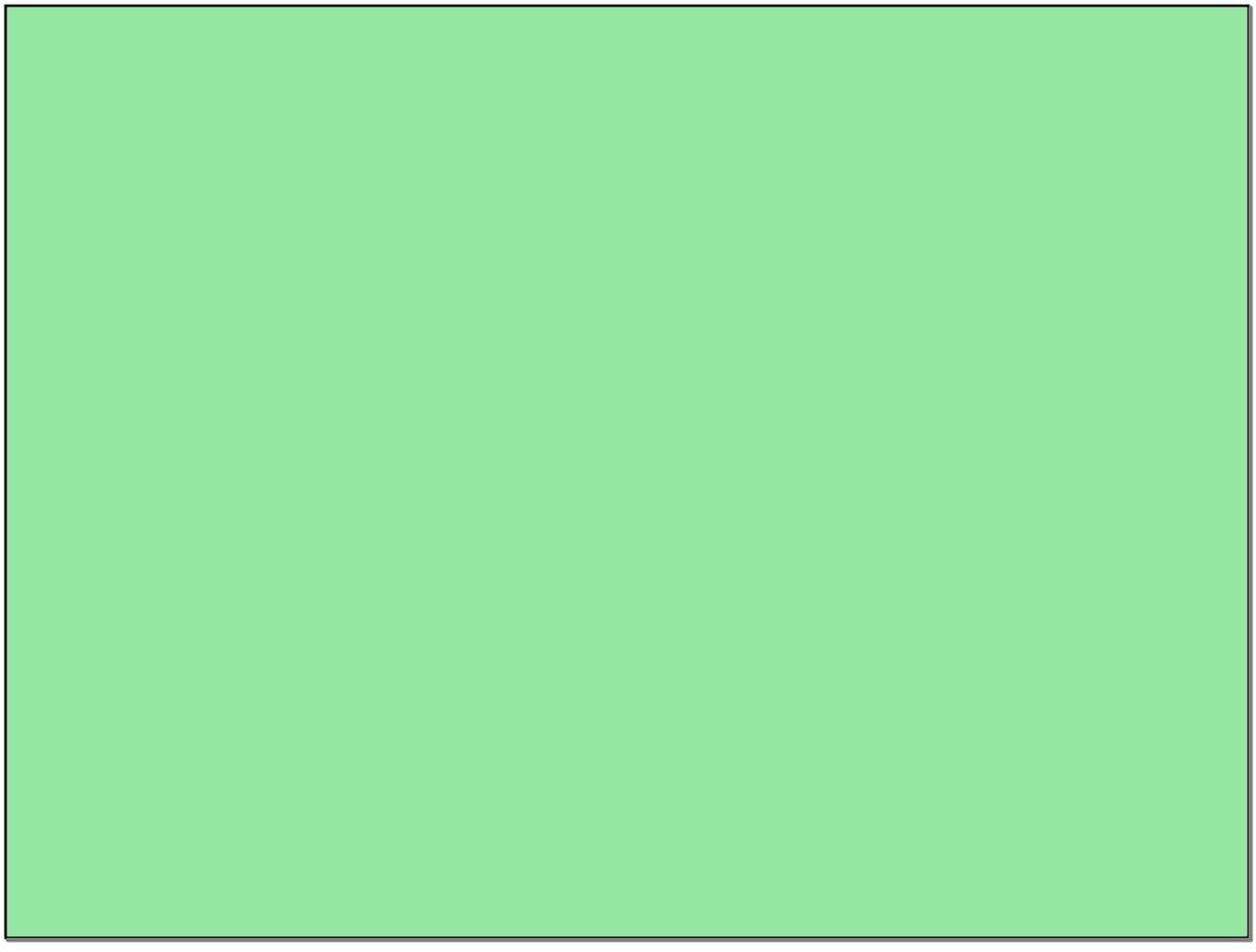

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