Semester Review

Inverse Trig Functions

Always in Radians!

$-\begin{cases}\cos^{-1}x\\ \sec^{-1}x\\ \cot^{-1}x\end{cases}$ All +

$\begin{matrix}\csc^{-1}x\\ \sin^{-1}x\\ \tan^{-1}x\end{matrix}\Big\}-$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ $\quad \pi/3$

$\sin \pi/6 = 1/2$

$\sin^{-1}(1/2) = \pi/6$

$\text{Arcsec}(-\sqrt{2}) = \frac{3\pi}{4}$ $\quad \pi/4$

$\tan\left(\text{Arccsc} -\frac{7}{2}\right)$ $\frac{r}{y}$ $\quad 3\sqrt{5}$, θ, 7, -2

$\tan(\theta) = \frac{y}{x} = \frac{-2}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{-2\sqrt{5}}{15}$

$x^2 + 4 = 49$

$\sqrt{x^2} = \sqrt{45}$

$= \pm 3\sqrt{5}$

$\cos\left(\sin^{-1}\frac{4}{x}\right)$ $\frac{y}{r}$ $\quad x$, 4, a, $\sqrt{x^2-16}$

$\cos(\theta) = \frac{x}{r}$

$= \frac{\sqrt{x^2-16}}{x}$

$a^2 + 16 = x^2$

$\sqrt{a^2} = \sqrt{x^2-16}$

19(f)

$\sin\left[\underbrace{\tan^{-1}(3/4)}_{A} - \underbrace{\cos^{-1}\left(\frac{-5}{13}\right)}_{B}\right]$

$\sin[A - B]$

$= \sin A \cos B - \cos A \sin B$

$\sin\left(2\,\underbrace{\text{Arctan } 3/4}_{\theta}\right)$ $\quad 5$, 3, θ, 4

$\sin(2\theta)$

$= 2\sin\theta\cos\theta$

$= \frac{2}{1}\left(3/5\right)\left(4/5\right)$

$= \frac{24}{25}$

~~Solve~~

$\cos^{-1}x + \cot^{-1}(\sqrt{3}) = \pi$ $\quad \pi/6$

$\cos^{-1}x + \pi/6 = \pi$
$\qquad\qquad -\pi/6$

$\cos^{-1}x = \frac{5\pi}{6}$ $\quad \pi/6$

$\cos\frac{5\pi}{6} = x$

$-\frac{\sqrt{3}}{2} = x$

Solving Inv Trig Eq.

1) Isolate trig func with a variable
2) Do inverse switch
3) Solve for variable.

Solving Trig Equations

$[0, 2\pi)$

$$\boxed{\begin{aligned}\tan\frac{x}{2} &= \sqrt{\frac{1-\cos x}{1+\cos x}} \\ &= \frac{\sin x}{1+\cos x} \\ &= \frac{1-\cos x}{\sin x}\end{aligned}}$$

21(d) $\tan\frac{x}{2} + 2\sin 2x = \csc x$

$$\sin x\left[\frac{1-\cos x}{\sin x} + \overset{\sin x}{4\sin x\cos x} = \frac{1}{\sin x}\right]$$

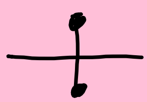
$$\cancel{1} - \cos x + 4\sin^2 x\cos x = \cancel{1}$$

$$-1 \qquad\qquad -1$$

$$4\sin^2 x\cos x - \cos x = 0$$

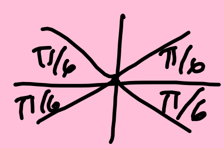
$$\cos x\,(4\sin^2 x - 1) = 0$$

$$\cos x = 0 \qquad 4\sin^2 x - 1 = 0$$



$$\sqrt{\sin^2 x} = \sqrt{1/4}$$

$$\sin x = \pm 1/2$$



$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Check:

1) variables in denom.
2) square both sides)

April 22, 2022

Polar Coord.

(r, θ)

$(6, 240^\circ)$

$x = 6\cos 240^\circ = 6 \cdot \frac{-1}{2} = -3$

$y = 6\sin 240^\circ = 6 \cdot \frac{-\sqrt{3}}{2} = -3\sqrt{3}$

$\boxed{(-3, -3\sqrt{3})}$

Rect. Coord

(x, y)

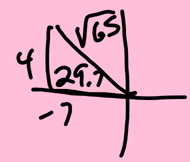
$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$

$\tan\theta = y/x$

$(-7, 4)$ Change to polar coord.



$16 + 49 = r^2$

$\sqrt{65} = \sqrt{r^2}$

$\tan\theta = \frac{4}{-7}$

$\theta = 150.3^\circ$ $\quad \tan^{-1}(-4/7) = 29.7^\circ$

$\boxed{(\sqrt{65}, 150.3^\circ)}$

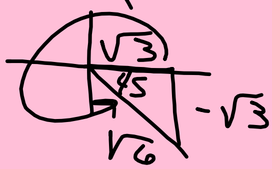
$$\frac{35(\cos 70^\circ + i\sin 70^\circ)}{7(\cos 10^\circ + i\sin 10^\circ)} = 5(\cos 60^\circ + i\sin 60^\circ)$$

$$(\overset{x}{\sqrt{3}} - \overset{\cancel{y}i}{i\sqrt{3}})^6 =$$

$$(\sqrt{3})^2 + (\sqrt{3})^2 = r^2$$

$$3 + 3 = r^2$$

$$\sqrt{6} = \sqrt{r^2}$$



$$\tan\theta = \frac{-\sqrt{3}}{\sqrt{3}}$$

$$= -1$$

$$\theta = 315^\circ$$

$$\left[\sqrt{6}(\cos 315^\circ + i\sin 315^\circ)\right]^6$$

$\cdot 6$

$$\sqrt{6}^6 (\cos 1890^\circ + i\sin 1890^\circ)$$

$$(6^{1/2})^6$$

$$6^3$$

$$216(\cos 1890^\circ + i\sin 1890^\circ)$$

$$360^\circ \cdot 5 = 1800^\circ$$

$$216(\cos 90^\circ + i\sin 90^\circ)$$

$$216(0 + i \cdot 1)$$

$$= \boxed{216i}$$

Solve for x.

$$(x^4)^{1/4} = (\sqrt{3} - i\sqrt{3})^{1/4}$$

$$x = \left[\sqrt{6}(\cos 315^\circ + i\sin 315^\circ)\right]^{1/4}$$

$$(6^{1/2})^{1/4} \qquad 315 \cdot \frac{1}{4} = 78.75$$

$$360^\circ \cdot \frac{1}{4} = 90^\circ$$

1) $6^{1/8}(\cos \underset{+90}{78.75} + i\sin 78.75^\circ)$

2) $6^{1/8} \operatorname{cis} 168.75^\circ$

3) $6^{1/8} \operatorname{cis} 258.75^\circ$

4) $6^{1/8} \operatorname{cis} 348.75^\circ$